K VALUES VARY WITHIN SPACE AND WITH DIRECTION

HETEROGENEITY - describes spatial variation

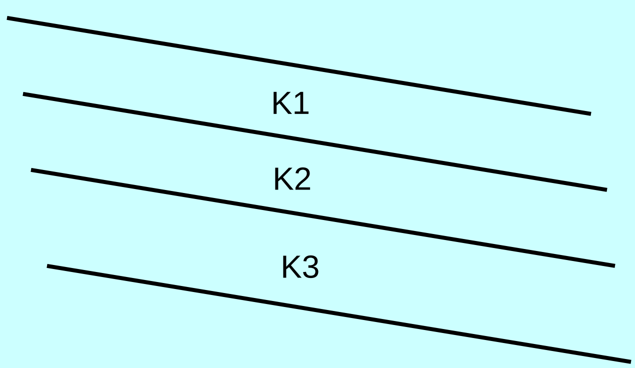
ANISOTROPY - describes directional variation

HOMOGENEOUS - uniform throughout (K independent of position)

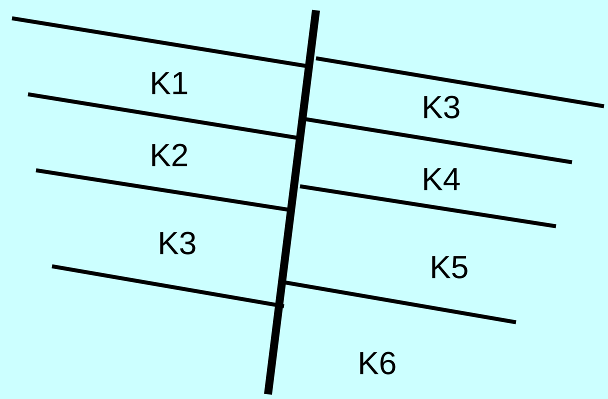
ISOTROPIC - properties do not vary with direction

LAYERED HETEROGENEITY

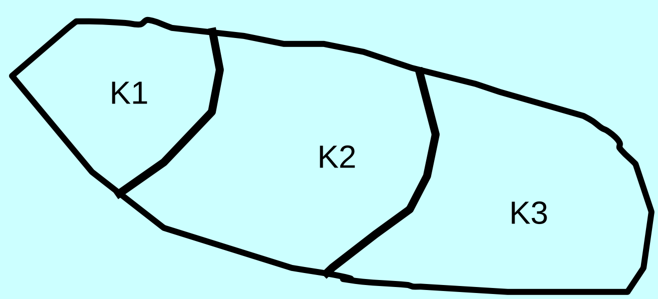
Individual layers are homogeneous



DISCONTINUOUS HETEROGENEITY
e.g. across a fault

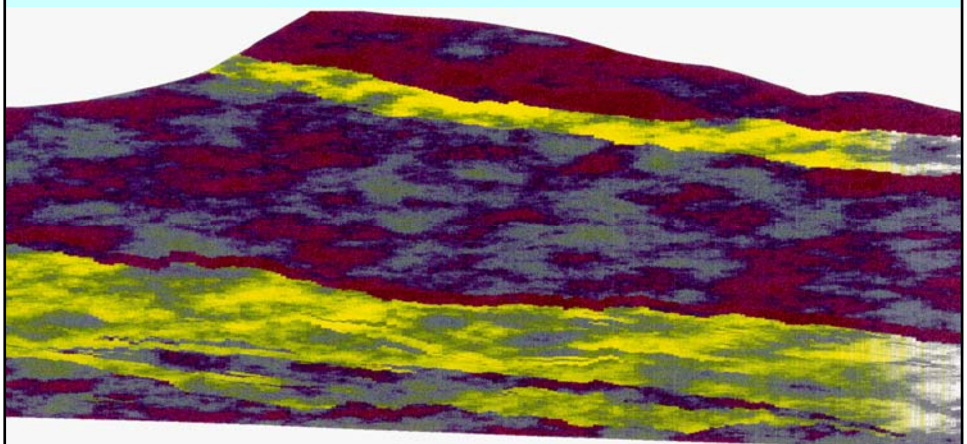


TRENDING HETEROGENEITY
variation in sedimentation patterns



RANDOM HETEROGENEITY

small scale variation with a structure that isn't easily tied to geologic process, although we know it is its basis we can describe with geostatistics (spatial statistics)



AVERAGING OPTIONS

Weighted Arithmetic

$$\frac{\sum K_i d_i}{\sum d_i}$$

Geometric

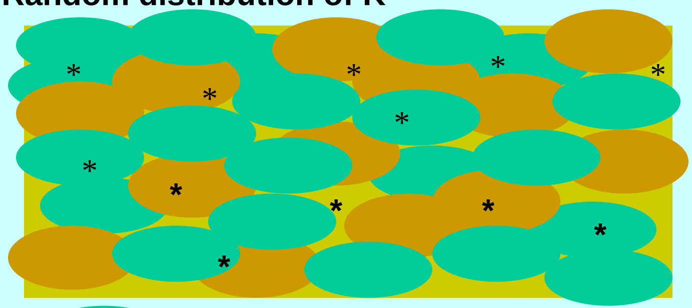
$$\sqrt[N]{K_1 K_2 \ldots K_N} \text{ or } 10^{\left(\frac{1}{N}\left(\log K_1 + \log K_2 + \ldots + \log K_N\right)\right)} \text{ or } 10^{\left(\frac{1}{N}\log\left(K_1 K_2 \ldots K_N\right)\right)}$$

Above assumes each K represents an equal volume, each could be weighted by the volume represented

Weighted Harmonic

$$\frac{\sum d_i}{\sum \frac{d_i}{K_i}}$$

Random distribution of K



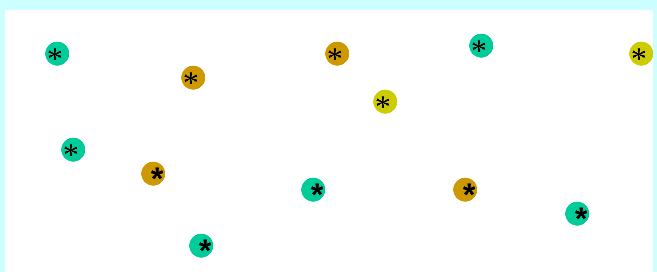
6 $1x10^{-3}$cm/s

4 $1x10^{-5}$cm/s

2 $1x10^{-6}$cm/s

*** sample locations assumed to be representative of the proportions**

Realize you would only know the values shown



You might have clues based on lithology

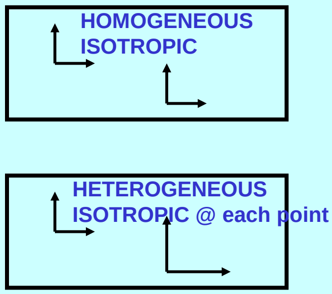
ANISOTROPY

CONSIDER K IN THE PRINCIPLE DIRECTIONS X, Y, Z

ISOTROPIC - $K_X = K_Y = K_Z$

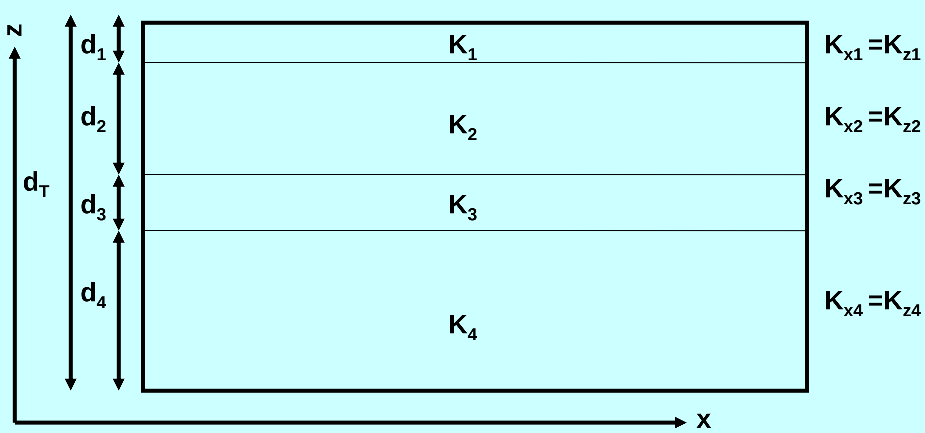
TRANSVERSELY ISOTROPIC - $K_X = K_Y$ not $= K_Z$

CONSIDER THE RELATIONS OF K IN A VERTICAL CROSS SECTION



THERE IS A RELATION BETWEEN LAYERED HETEROGENEITY & ANISOTROPY

an equivalent K can be calculated to simplify complex systems thus making it possible to apply Darcy's Law



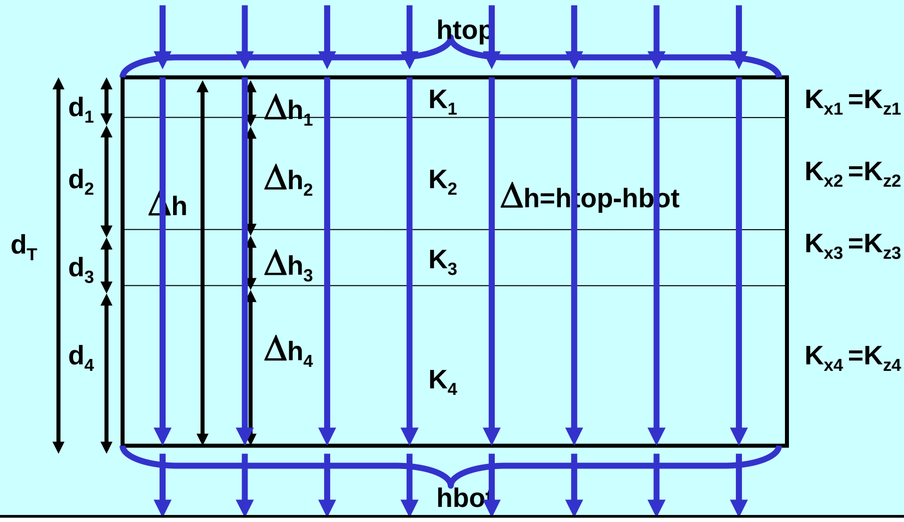
Consider Flow Perpendicular to Layering:

Say that $K_4 = K_1 < K_3 << K_2$

$V = Q/A$ $Q_{in} = Q_{out}$ therefore $V_{in} = V_{out}$

Δh_i = head drop across layer i

$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 + \Delta h_4 = htop - hbot$



By Darcy's Law:

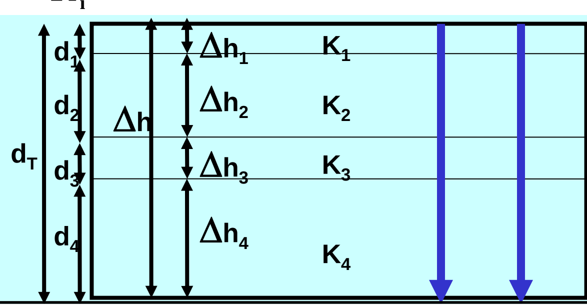
$$V = \frac{K_1 \Delta h_1}{d_1} = \frac{K_2 \Delta h_2}{d_2} = \ldots \frac{K_n \Delta h_n}{d_n} = \frac{K_{eq} \Delta h}{d_T}$$

Rearrange: Expand head difference term: Substitute Darcy's Law for head differences:

$$K_{eq} = \frac{V d_T}{\Delta h} = \frac{V d_T}{\Delta h_1 + \Delta h_2 + \ldots + \Delta h_n} = \frac{V d_T}{\frac{V d_1}{K_1} + \frac{V d_2}{K_2} + \ldots + \frac{V d_n}{K_n}}$$

$$K_{eq} = \frac{d_T}{\sum \frac{d_i}{K_i}}$$

Equivalent K for flow perpendicular to layers

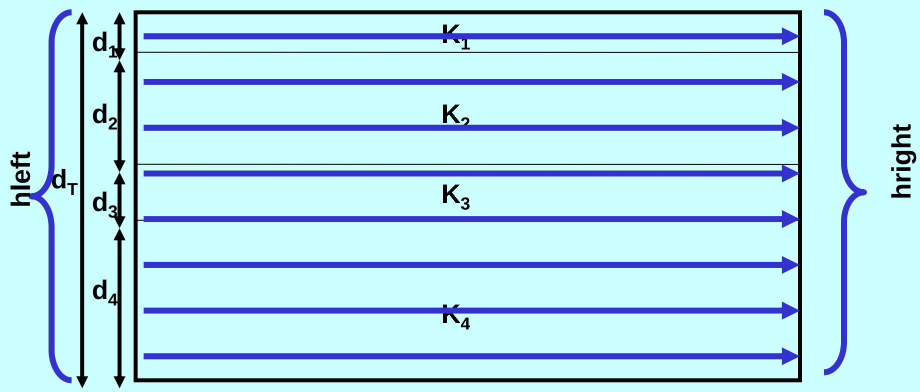


Consider Flow Parallel to Layering:

Say for example $K_4 = K_1 < K_3 << K_2$

$V = Q/A$ $Q_{in} = Q_{out}$ therefore $V_{in} = V_{out}$

Δh = hleft-hright



By Darcy's Law:

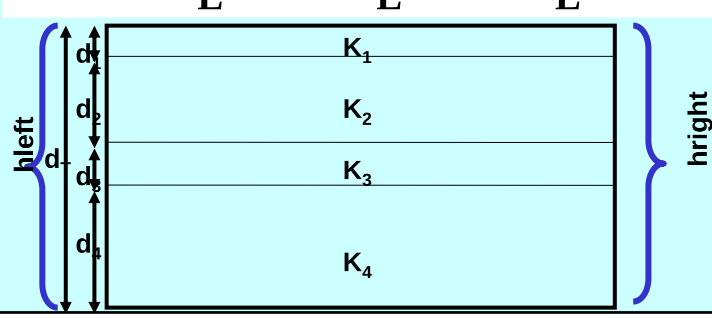
$$Q_1 = K_1 \frac{\Delta h}{L} d_1 (1) \qquad Q_2 = K_2 \frac{\Delta h}{L} d_2 (1)$$

$$Q_3 = K_3 \frac{\Delta h}{L} d_3 (1) \qquad Q_4 = K_4 \frac{\Delta h}{L} d_4 (1)$$

1 for unit width into the paper

Total flow is their sum:

$$Q_T = K_1 d_1 \frac{\Delta h}{L} + K_2 d_2 \frac{\Delta h}{L} + K_3 d_3 \frac{\Delta h}{L} + K_4 d_4 \frac{\Delta h}{L}$$



$$V = \frac{Q}{A} = \frac{Q}{d_T(1)}$$

1 for unit width into the paper

Substitute Eqtn for Q from previous slide

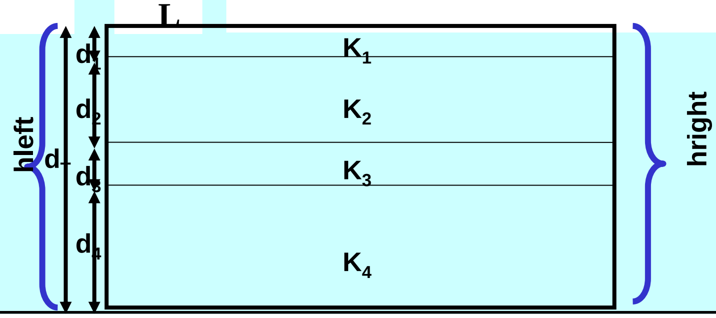
Simplify with a summation

$$V = \frac{K_1 d_1 \frac{\Delta h}{L} + K_2 d_2 \frac{\Delta h}{L} + K_3 d_3 \frac{\Delta h}{L} + K_4 d_4 \frac{\Delta h}{L}}{d_T} = \frac{\sum (K_i d_i)\Delta h}{d_T L}$$

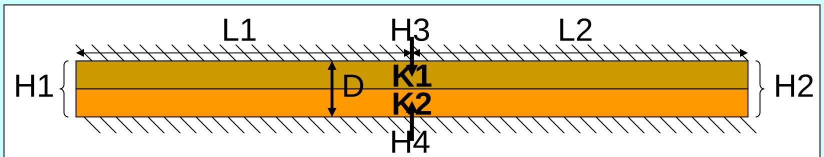
We can divide V by gradient to get K:

Cancel the gradients:

$$K_{eq} = \frac{V}{i} = \frac{V}{\frac{\Delta h}{L}} = \frac{\sum_{i=1}^{n} K_i d_i}{d_T}$$

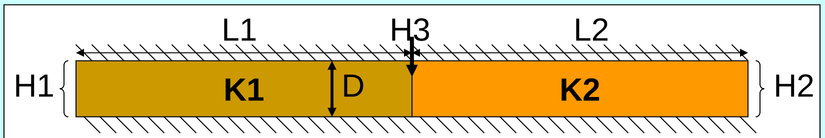


Calculate Flow and Heads between boundaries



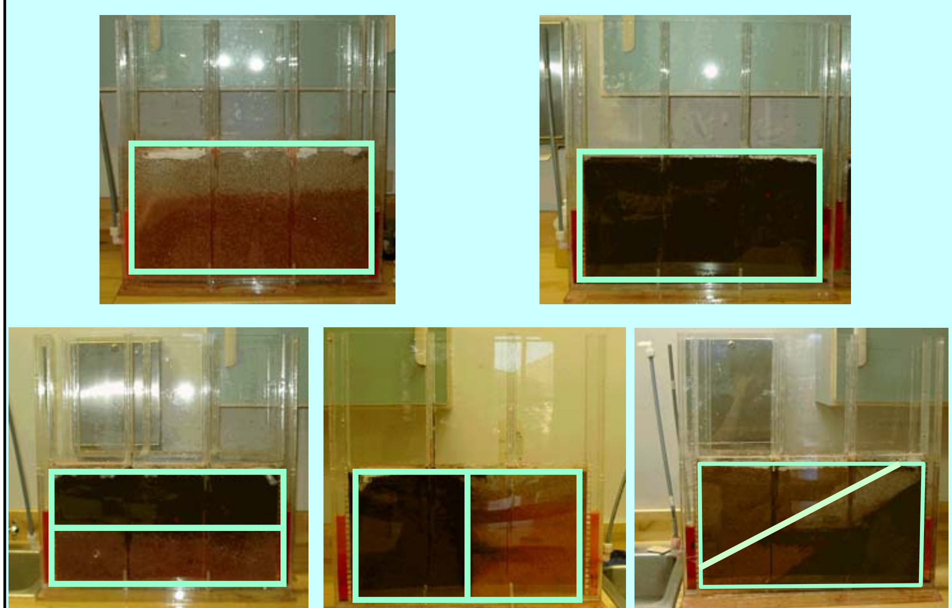
H1 = 20cm
H2 = 10cm
K1 = 1cm/sec
K2 = 0.2cm/sec
L1 = 30cm
L2 = 30cm
D = 2cm
Q @ H2 = ??
H3 = ??, H4 = ??

Calculate Flow and Heads between boundaries



H1 = 20cm
H2 = 10cm
K1 = 1cm/sec
K2 = 0.2cm/sec
L1 = 30cm
L2 = 30cm
D = 2cm
Q @ H2 = ??
H3 = ??

Calculate Equivalent K's of the "Ant Farms"



Looking Back

AVERAGING OPTIONS

Recall the options and when each is appropriate

Weighted Arithmetic

$$\frac{\sum K_i d_i}{\sum d_i}$$

Geometric

$$\sqrt[N]{K_1 K_2 \ldots K_N} \text{ or } 10^{\left(\frac{1}{N}\left(\log K_1 + \log K_2 + \ldots + \log K_N\right)\right)} \text{ or } 10^{\left(\frac{1}{N}\log\left(K_1 K_2 \ldots K_N\right)\right)}$$

Above assumes each K represents an equal volume, each could be weighted by the volume represented e.g. each K (or for the middle option log(K)) would be multiplied by d_i/d_{min} & then N= sum of d_i/d_{min}

Weighted Harmonic

$$\frac{\sum d_i}{\sum \frac{d_i}{K_i}}$$

$$V_{Darcy} = -K\left(\frac{h_2 - h_1}{l}\right)$$

K is equivalent K over the distance l

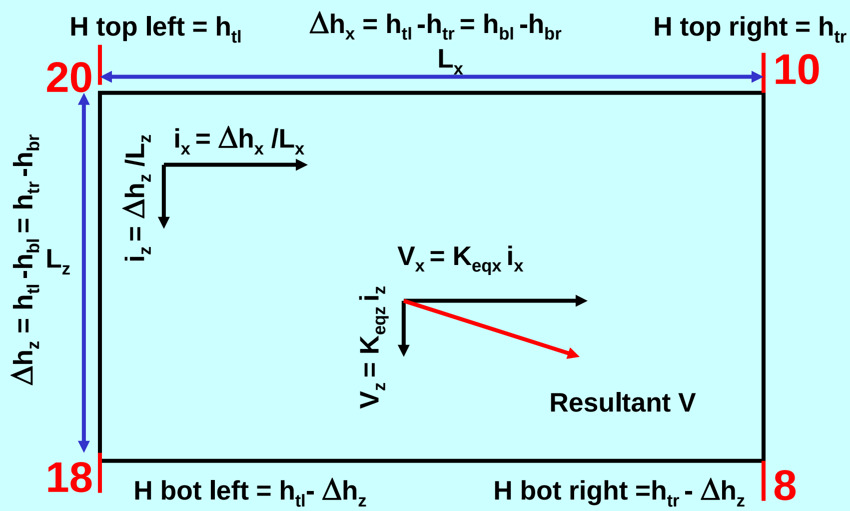
l is the flow path distance between h2 and h1

gradient is in the direction of flow

when using velocity to calculate volumetric discharge, area is perpendicular to the gradient

Consider a Diagonal Gradient

Given a K_{eqx} and a K_{eqz}



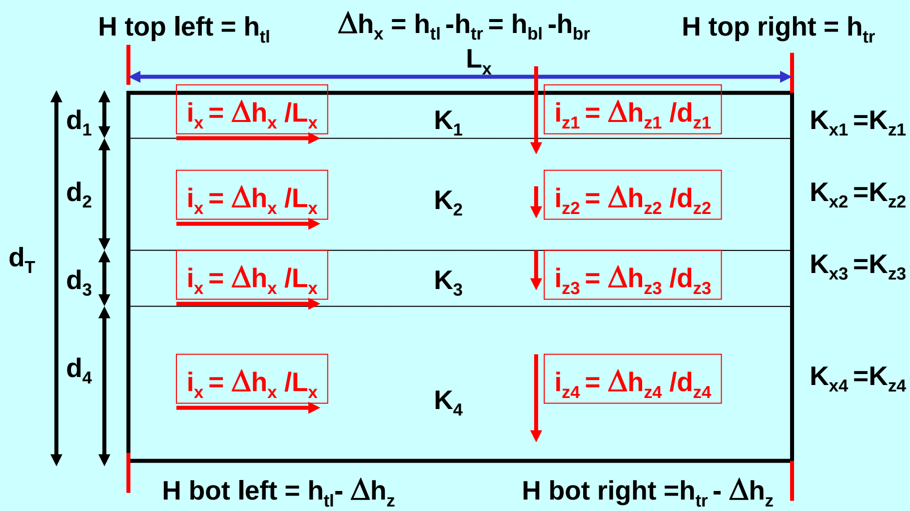
Calculate a resultant velocity at the center

now the gradient is different at top and bottom and from side to side

$Kx=1x10^{-5}$ m/s $Ky=1x10^{-6}$m/s

Consider a Gradient Askew to Layering:

The overall velocity is actually composed of individual velocity vectors in each layer

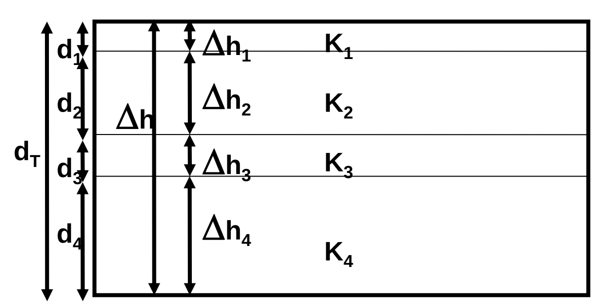


To calculate Δh_{zi}:

$$V = \frac{K_1 \Delta h_1}{d_1} = \frac{K_2 \Delta h_2}{d_2} = \ldots \frac{K_n \Delta h_n}{d_n} = \frac{K_{eq} \Delta h}{d_T} = \frac{\frac{d_T}{\sum \frac{d_i}{K_i}} \Delta h}{d_T} = \frac{\Delta h}{\sum \frac{d_i}{K_i}}$$

so $$\frac{K_i \Delta h_i}{d_i} = \frac{\Delta h}{\sum \frac{d_i}{K_i}}$$

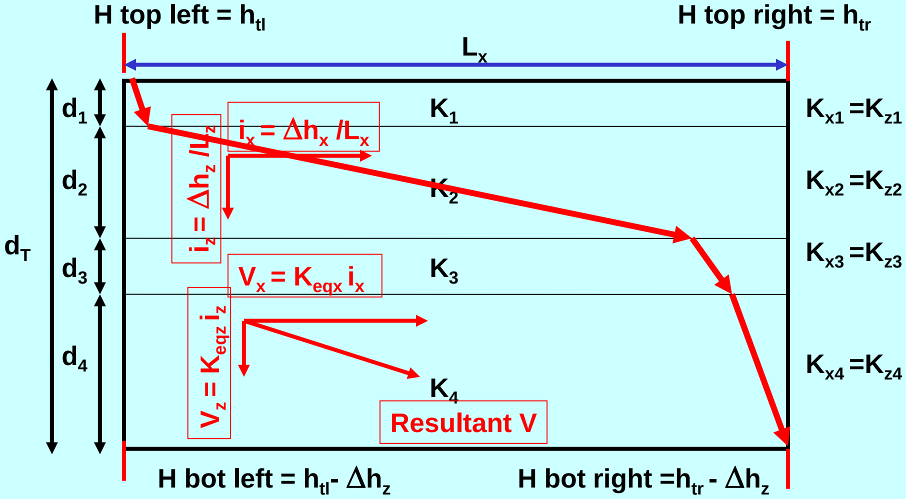
$$\Delta h_i = \frac{\Delta h d_i}{K_i \sum \frac{d_i}{K_i}}$$



Consider a Gradient Askew to Layering:

The overall velocity is actually composed of individual velocity vectors in each layer

$K_4 = K_1 < K_3 << K_2$



Refraction at Layer Interfaces

$$\frac{K_1}{K_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$K_4 = K_1 < K_3 << K_2$