Superposition Applications

applicable to linear conditions
(i.e. confined or unconfined if drawdown << aquifer thickness s<<b)

Utility of superposition

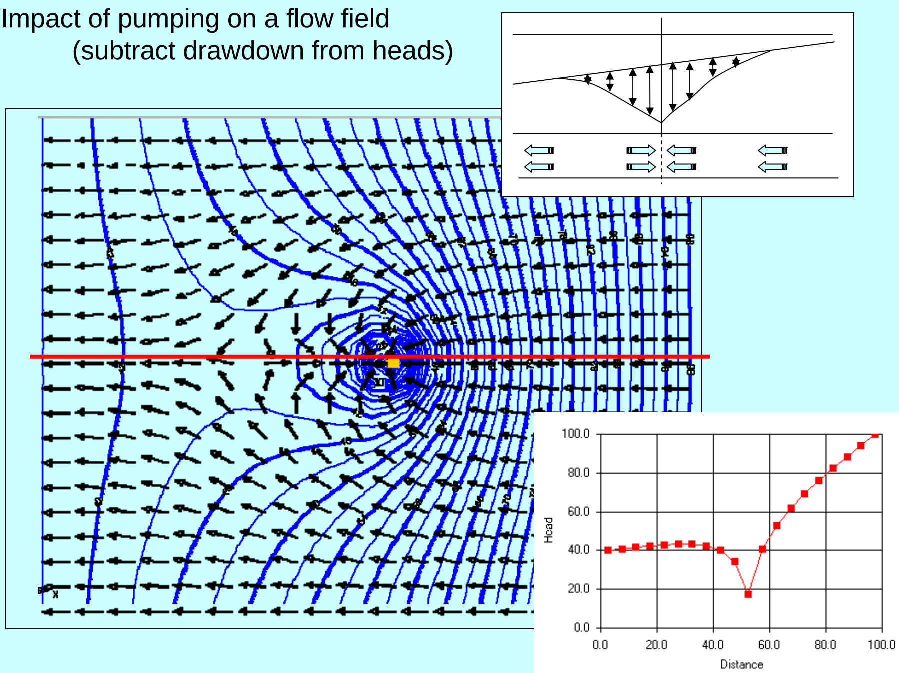
* Impact of pumping on a flow field

* Pumping from a number of wells

* Boundaries by Image well theory

* Incremental Pumping

•Impact of pumping on a flow field
(subtract drawdown from heads)

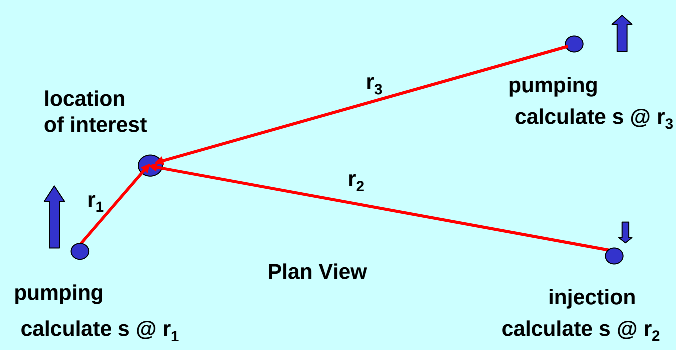


Drawdown from Pumping a Number of Wells

Superposed Solutions

*Pumping from a number of wells

$$s = \frac{Q_1}{4\pi T}W(u_1) + \frac{Q_2}{4\pi T}W(u_2) + \ldots\ldots$$ where: $$u_i = \frac{r_i^2 S}{4Tt_i} \qquad i = 1, 2, \ldots.$$



sum s_1 from Q_1@ r_1 s_2 from Q_2@ r_2 (note Q_2 is negative) s_3 from Q_3@ r_3 ….. etc etc ….. yields total s at observation well

Let's try it

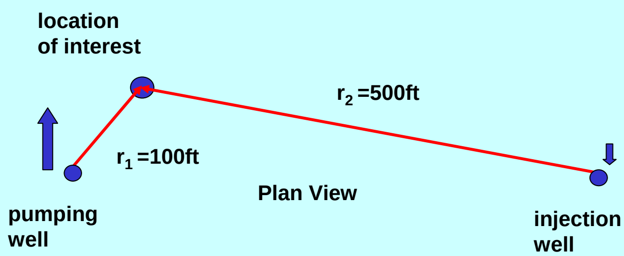
T= $1x10^0$ ft^2 /day

S = $1x10^{-5}$

Q @ r_1 = 40 ft^3 /day

Q @ r_2 = -40 ft^3 /day

What is the drawdown at the location of interest after 10 days?



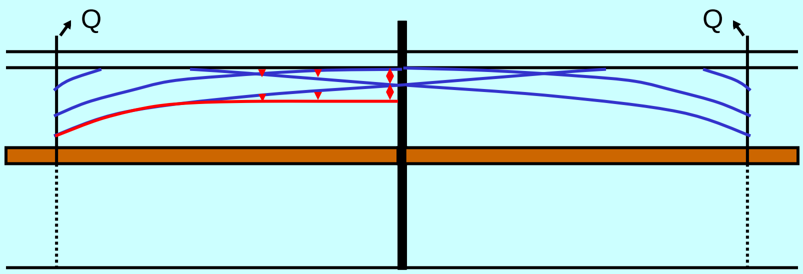
NEXT

Influence of Boundaries on Drawdown

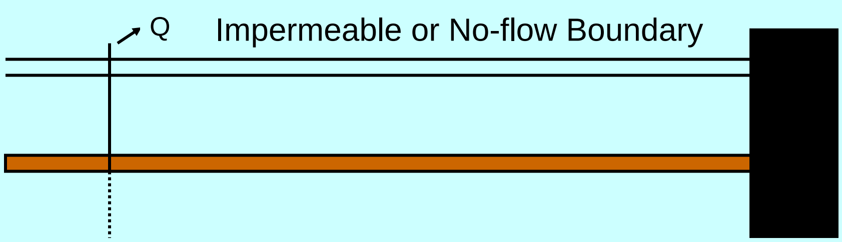
Superposed Solutions
using
Image Wells

•Image well theory

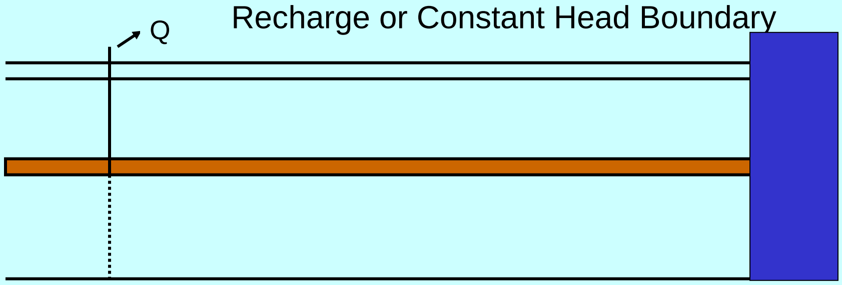
add image well drawdown from actual well drawdown



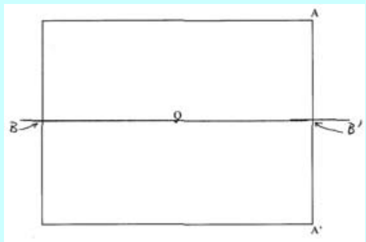
Click here to visualize well interference
http://www.mines.edu/~epoeter/_GW/15wh4Superpsition/WellInterference.html



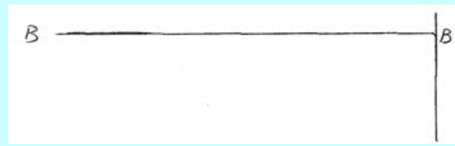
No aquifer is infinite. How will boundaries affect response?



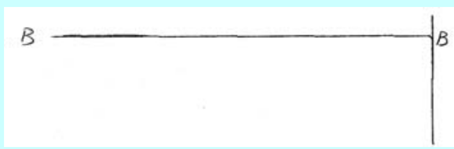
For the following situation with pumping well Q make your qualitative estimates of the relative drawdown.



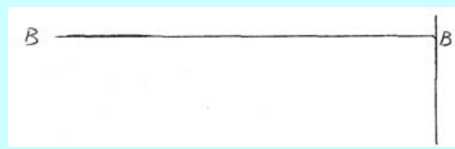
Sketch the cone of depression due to pumping of well Q assuming A-A' is a no flow boundary

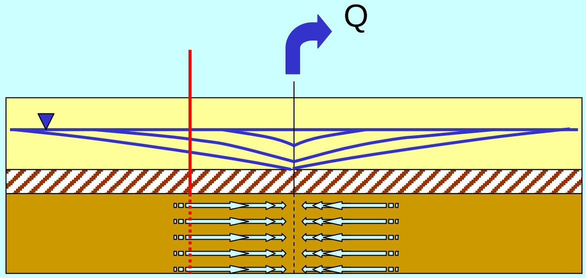


Sketch the cone of depression due to pumping of well Q assuming the aquifer is infinite

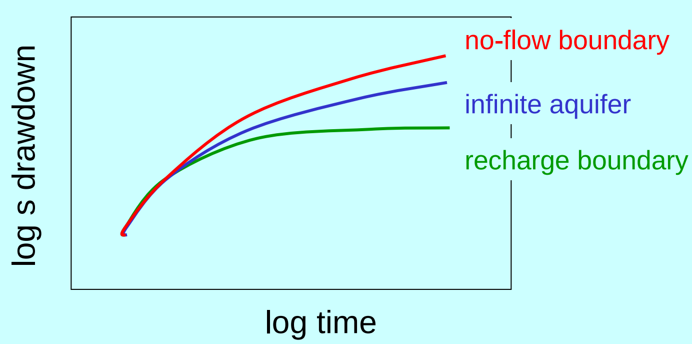


Sketch the cone of depression due to pumping of well Q assuming A-A' is a constant head boundary

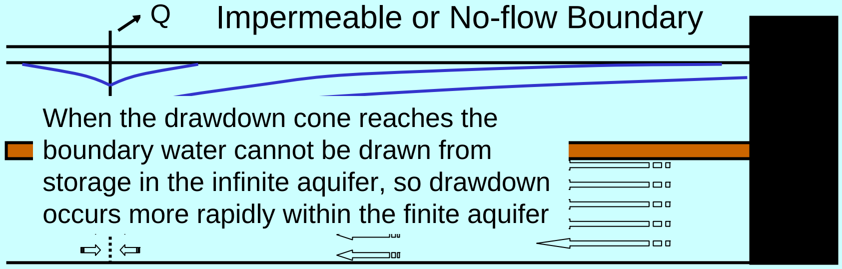


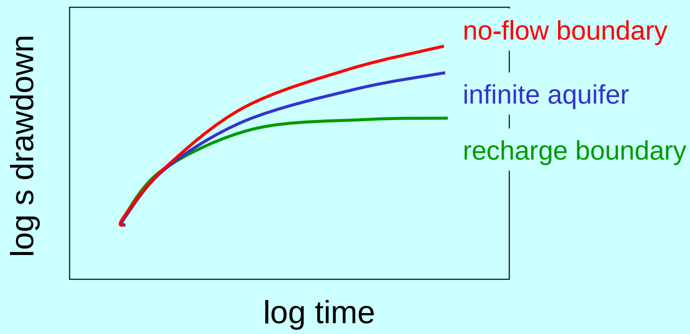


at the red observation well …..

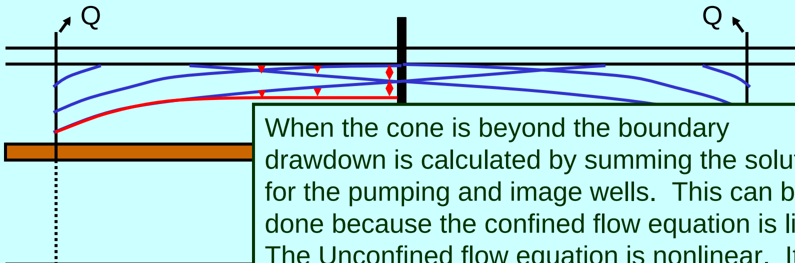


Impermeable or No-flow Boundary





Impermeable or No-flow Boundary



When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown
by creating a mathematical no-flow boundary

NO-FLOW = NO GRADIENT

So if we place an imaginary well

of equal strength

at equal distance across the boundary

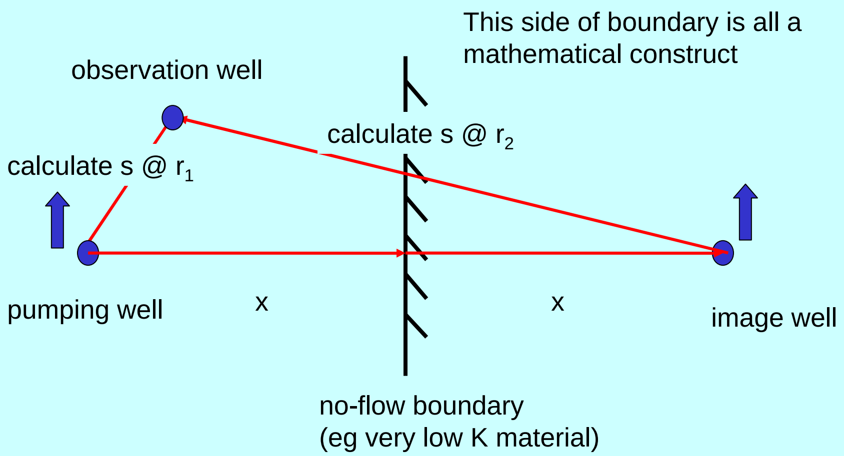
And superpose the solutions

We will have

equal drawdown, therefore equal head at the boundary, hence NO GRADIENT

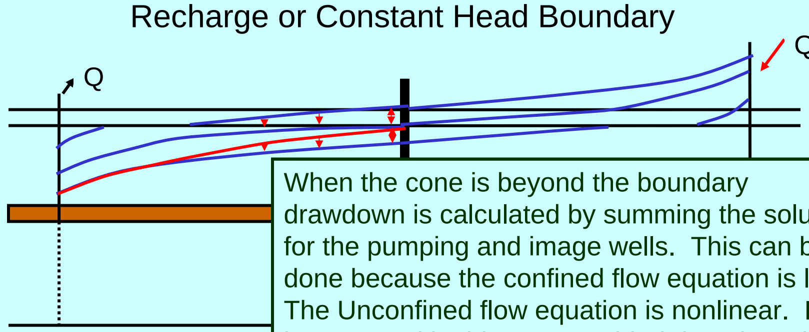
Let's look at it

Plan View



sum s @ r_1 and s @ r_2 drawdown is greater than without the boundary

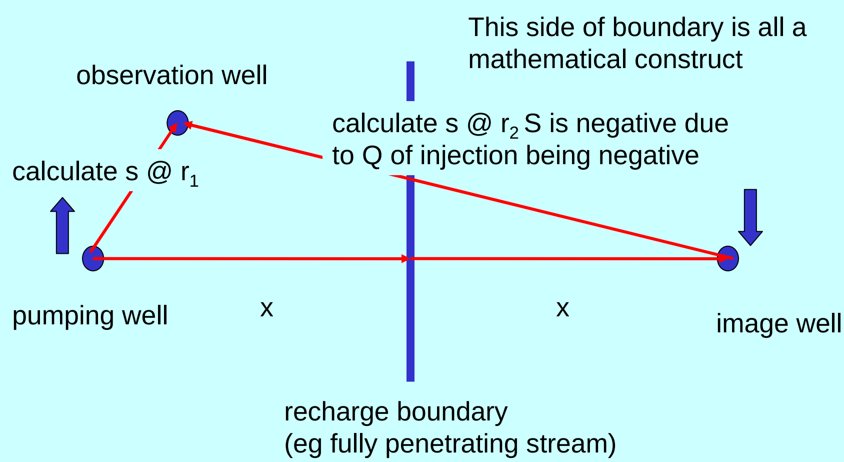
Recharge or Constant Head Boundary



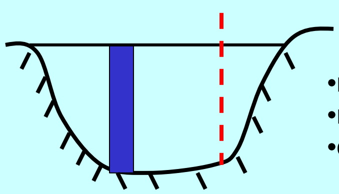
When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown
by creating a mathematical constant head boundary
CONSTANT HEAD = NO CHANGE IN HEAD
So if we place an imaginary well
of equal strength but opposite sign
at equal distance across the boundary
And superpose the solutions
We will have
equal but opposite drawdown, therefore NO HEAD CHANGE
Let's look at it

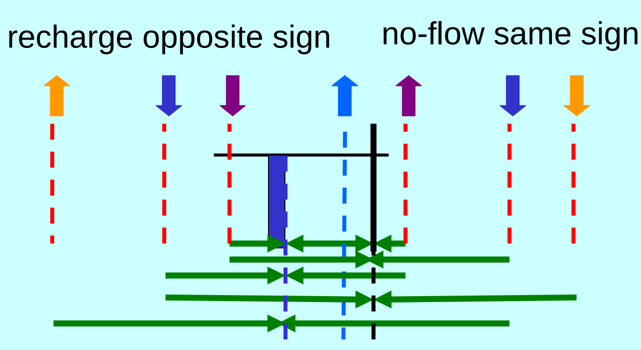
Plan View



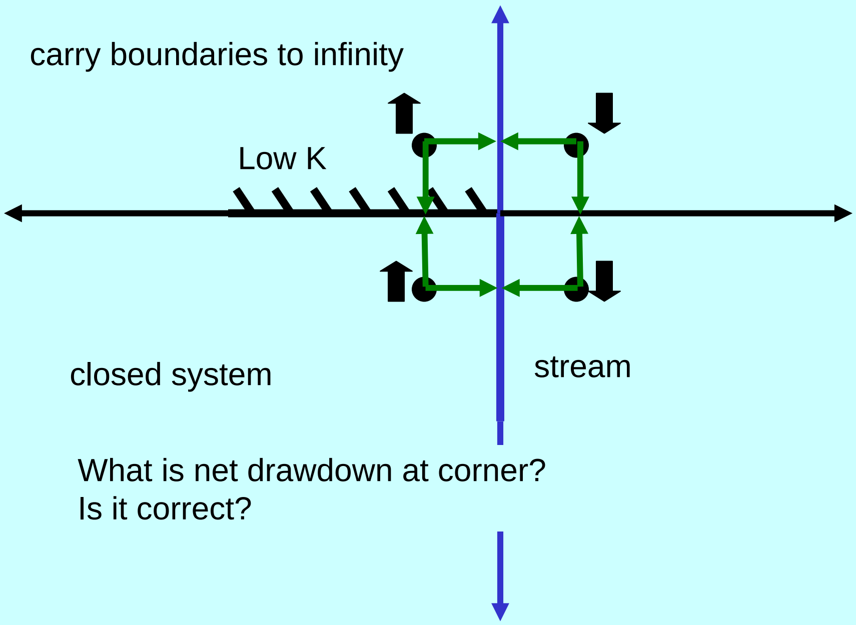
sum s @ r_1 and s @ r_2 drawdown is less than without the boundary



- multiple boundaries require multiple reflections
- reflect image wells also
- each boundary should be drawn to infinity and all reflections made



etc etc until the addition is insignificant at the time of interest carry boundaries to infinity



closed system

What is net drawdown at corner?
Is it correct?

NEXT

Influence of Changing Pumping Rate on Drawdown

Superposed Solutions in Time

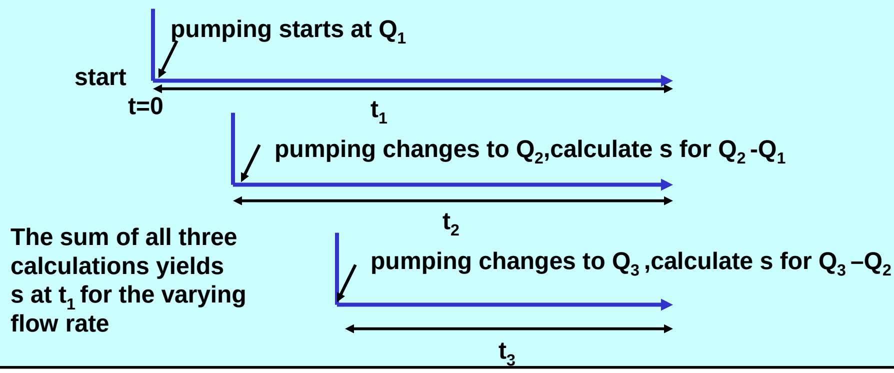
* Incremental Pumping

$$s = \frac{Q_1}{4\pi T} W(u_1) + \frac{\Delta Q_2}{4\pi T} W(u_2) + \frac{\Delta Q_3}{4\pi T} W(u_3) \;\ldots\ldots\ldots$$

Q_1 = initial rate — u_1 for t since pumping started, $\mathbf{t_1}$

$\Delta Q_2 = Q_2 - Q_1$ — u_2 for t since incremented rate, $\mathbf{t_2}$

$\Delta Q_3 = Q_3 - Q_2$ — u_3 for t since second increment, $\mathbf{t_3}$



Let's try it

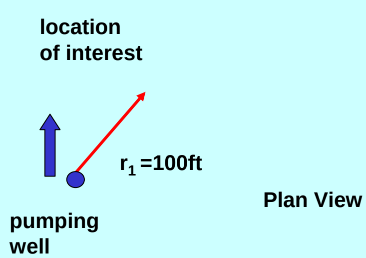
T= 1x10^0 ft^2 /day

S = 1x10^{-5}

Q = 40 ft^3 /day for 5 days

Q = 10 ft^3 /day for the next 5 days

What is the drawdown at the location of interest after 10 days?

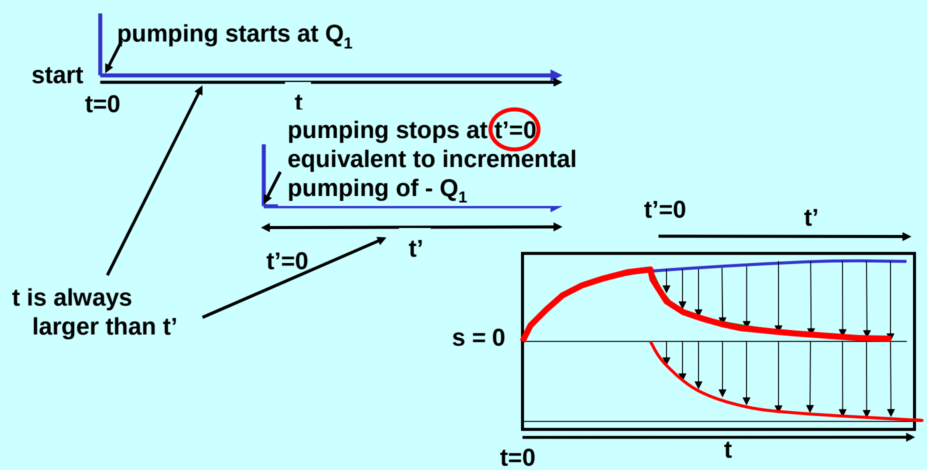


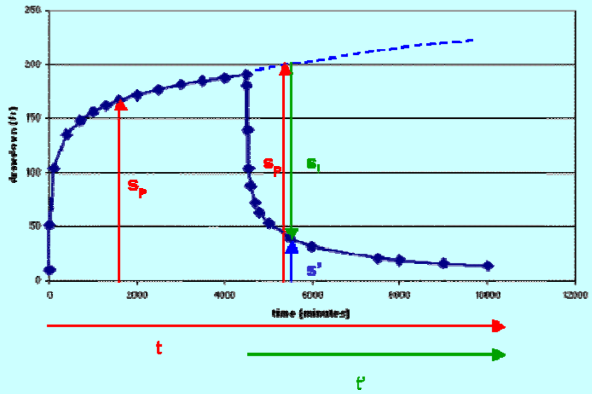
NEXT

We can use this to analyze aquifer test recovery data

•Using incremental drawdown, we can analyze aquifer test recovery data by adding drawdown from injection of –Q at the time when the pump is shut off

$$s' = \frac{Q}{4\pi T}W(u) - \frac{Q}{4\pi T}W(u') \quad \textbf{for} \quad u = \frac{r^2 S}{4Tt} \qquad u' = \frac{r^2 S}{4Tt'}$$





s' = residual drawdown

s_p = drawdown from pumping

t = time since pumping began

s_i = drawdown from "injection"

t' = time since pumping stopped

OR for small u (small r, long t)

$$s' = \frac{2.3Q}{4\pi T}\left[\log\frac{2.25Tt}{r^2S} - \log\frac{2.25Tt'}{r^2S}\right] = \frac{2.3Q}{4\pi T}\log\frac{t}{t'}$$

plot of s' vs log (t/t') will be a straight line

$$T = \frac{2.3Q}{4\pi\Delta s}$$

Δs over one log cycle t/t'

and

If data are from an observation well,
S is obtained from the value of s & t when pumping stopped, see next slide

If data are from an observation well, then S can be estimated by:

1. identifying the value of s at the end of the test
2. rearranging the Theis equation

$$s = \frac{Q}{4\pi T} W(u) \qquad W(u) = \frac{s4\pi T}{Q}$$

and solving for W(u) using the Q that prevailed during the test

3. finding u from a table of u vs. W(u)
4. rearranging the expression for u

$$u = \frac{r^2 S}{4Tt} \qquad S = \frac{4Ttu}{r^2}$$

and solving for S using the t at the end of the test corresponding to the s at the end of the test

Try a recovery analysis

(and compare to Theis results)

using:

http://www.mines.edu/~epoeter/_GW/15wh4Superpsition/recovery-classex.xls

$Q = 1925\ ft^3/day$

R = 50 ft