ALERT! ALERT! CORRECTION TO LAST LECTURE

Suppose that source enters the up gradient end of a column
At a continuous concentration of C_o=1000mg/l
K = 0.1 cm/sec
dh = 10 cm
dl = 100 cm
ϕ = 0.2
Dispersivity α_x = 5 cm
What will the concentration be at 50 cm after 1000sec?

average linear velocity

$$\bar{v} = \frac{Kdh}{\phi dl} = \frac{0.1\frac{cm}{\sec}}{0.2}\frac{10cm}{100cm} = 0.05\frac{cm}{\sec}$$

distance traveled in 1000sec?

$$d = \bar{v}t = 0.05\frac{cm}{\sec}1000\sec = 50cm$$

By inspection we expect the concentration should be 0.5*C_o=500mg/l
But let's carry out the calculation

ALERT! ALERT! CORRECTION TO LAST LECTURE – EXCEL WAS CORRECT

$$\bar{v} = 0.05\frac{cm}{\sec} \qquad D_x = \bar{v}\alpha + D^* = 0.05\frac{cm}{\sec}5cm + 1x10^{-10}\frac{m^2}{\sec}\frac{10000cm^2}{1m^2} = 0.25\frac{cm^2}{\sec}$$

$$C = \frac{C_o}{2}\left(erfc\left(\frac{x - vt}{2\sqrt{D_x t}}\right) + \exp\left(\frac{\bar{v}x}{D_x}\right)erfc\left(\frac{x+vt}{2\sqrt{D_x t}}\right)\right)$$

Often this term is ignored because it is typically small, but we carry it through here.

In this case including the second term provides a more accurate result

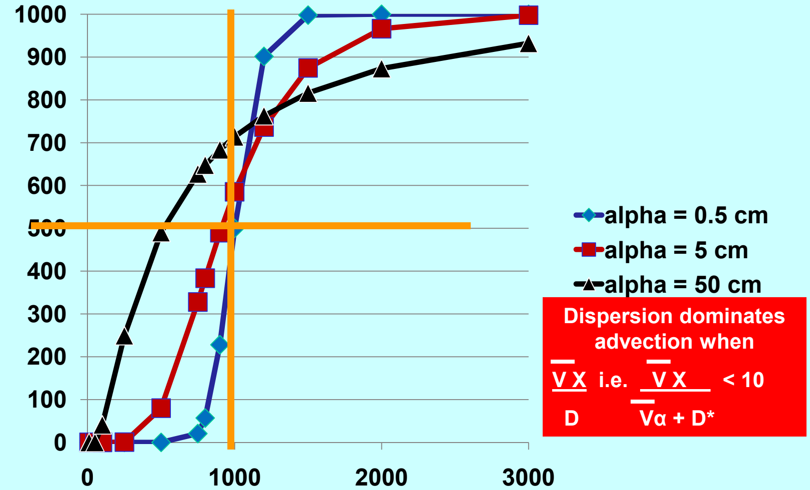
$$C = \frac{1000\frac{mg}{l}}{2}\left(erfc\left(\frac{50cm - 0.05\frac{cm}{\sec}1000\sec}{2\sqrt{0.25\frac{cm^2}{\sec}1000\sec}}\right) + \exp\left(\frac{0.05\frac{cm}{\sec}50cm}{0.25\frac{cm^2}{\sec}}\right)erfc\left(\frac{50cm + 0.05\frac{cm}{\sec}1000\sec}{2\sqrt{0.25\frac{cm^2}{\sec}1000\sec}}\right)\right)$$

$$C = 500\frac{mg}{l}\left(erfc\left(\frac{50cm - 50cm}{31.62cm}\right) + \exp\left(\frac{2.5\frac{cm^2}{\sec}}{0.25\frac{cm^2}{\sec}}\right)erfc\left(\frac{50cm+50cm}{31.62cm}\right)\right)$$

$$C = 500\frac{mg}{l}(erfc(0.0) + \exp(10)erfc(3.1623)) = 500\frac{mg}{l}(1.0 + 22026.46(7.743x10^{-7})) = 585.3\frac{mg}{l}$$

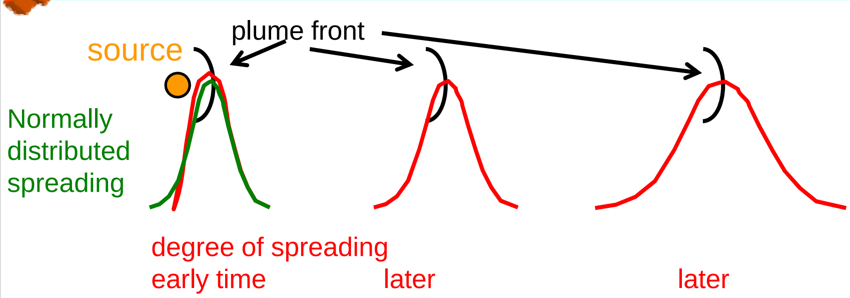
C is greater than 500 at the average travel time distance because dispersion dominates velocity near the source

http://inside.mines.edu/~epoeter/_GW/22ContamTrans/C1d.xls



CONCLUSION DOES NOT CHANGE

The second term is important for calculating C @ early times near the source.



Early time is not normally distributed because the mechanical dispersion does not move contaminants backwards so the contaminants must travel some distance from the source before the normal distribution is realized.

NOTE: In fact in heterogeneous environments (which means everywhere although more pronounced some places than others) contaminants often lag behind in low K materials biasing the spreading and causing long late tails

Suppose a source continuously enters that uniform flow field
With an initial concentration of C_o=1,000mg/l

pause to consider relationship of mass and concentration
Mass = Conc * Volume
Mass/Time = Conc * Velocity * Area = Conc * Q **(Q is discharge)**
Mass = Conc * Q * Time

Envision the source is submerged and
emanates from a 0.5cm high x 1cm wide zone
pause to consider the character of the source geometry

v = 0.05 cm/sec
dispersivity α_x = 5 cm
dispersivity α_y = 1/5 α_x
dispersivity α_z = 1/10 α_x

What will the concentration be at 50 cm
directly down gradient after 1000sec?
pause to consider the coordinate system

$$\bar{v} = 0.05\frac{cm}{\sec}$$

$$D_x = \bar{v}\alpha_x + D^* = 0.05\frac{cm}{\sec}5cm + 1x10^{-10}\frac{m^2}{\sec}\frac{10000cm^2}{1m^2} = 0.25\frac{cm^2}{\sec}$$

$$D_y = \bar{v}\alpha_x\frac{1}{5} + D^* = 0.05\frac{cm}{\sec}5cm\frac{1}{5} + 1x10^{-10}\frac{m^2}{\sec}\frac{10000cm^2}{1m^2} = 0.05\frac{cm^2}{\sec}$$

$$D_z = \bar{v}\alpha_x\frac{1}{10} + D^* = 0.05\frac{cm}{\sec}5cm\frac{1}{10} + 1x10^{-10}\frac{m^2}{\sec}\frac{10000cm^2}{1m^2} = 0.025\frac{cm^2}{\sec}$$

$$C(x,y,z,t) = \frac{C_o}{8}\left(erfc\left(\frac{x-\bar{v}_x t}{2\sqrt{D_x t}}\right)\right)\left(erf\left(\frac{y+\frac{Y}{2}}{2\sqrt{D_y\frac{x}{\bar{v}}}}\right) - erf\left(\frac{y-\frac{Y}{2}}{2\sqrt{D_y\frac{x}{\bar{v}}}}\right)\right)\left(erf\left(\frac{z+\frac{Z}{2}}{2\sqrt{D_z\frac{x}{\bar{v}}}}\right) - erf\left(\frac{z-\frac{Z}{2}}{2\sqrt{D_z\frac{x}{\bar{v}}}}\right)\right)$$

$$2\sqrt{D_x t} = 2\sqrt{0.25\frac{cm^2}{\sec}1000\sec} = 31.62cm$$

$$2\sqrt{D_y\frac{x}{\bar{v}}} = 2\sqrt{0.05\frac{cm^2}{\sec}\frac{50cm}{0.05\frac{cm}{\sec}}} = 14.14cm$$

$$2\sqrt{D_z\frac{x}{\bar{v}}} = 2\sqrt{0.025\frac{cm^2}{\sec}\frac{50cm}{0.05\frac{cm}{\sec}}} = 10cm$$

$$C(x,y,z,t)=\frac{C_o}{8}\left(erfc\left(\frac{x-\bar{v}_x t}{2\sqrt{D_x t}}\right)\right)\left(erf\left(\frac{y+\frac{Y}{2}}{2\sqrt{D_y\frac{x}{\bar{v}}}}\right)-erf\left(\frac{y-\frac{Y}{2}}{2\sqrt{D_y\frac{x}{\bar{v}}}}\right)\right)\left(erf\left(\frac{z+\frac{Z}{2}}{2\sqrt{D_z\frac{x}{\bar{v}}}}\right)-erf\left(\frac{z-\frac{Z}{2}}{2\sqrt{D_z\frac{x}{\bar{v}}}}\right)\right)$$

$$2\sqrt{D_x t}=2\sqrt{0.25\frac{cm^2}{\sec}1000\sec}=31.62cm$$

$$2\sqrt{D_y\frac{x}{\bar{v}}}=2\sqrt{0.05\frac{cm^2}{\sec}\frac{50cm}{0.05\frac{cm}{\sec}}}=14.14cm$$

$$2\sqrt{D_z\frac{x}{\bar{v}}}=2\sqrt{0.025\frac{cm^2}{\sec}\frac{50cm}{0.05\frac{cm}{\sec}}}=10cm$$

$$x-\bar{v}_x t=50cm-0.05\frac{cm}{\sec}1000\sec=0cm$$

$$y+\frac{Y}{2}=0+\frac{1cm}{2}=0.5cm \quad y-\frac{Y}{2}=0-\frac{1cm}{2}=-0.5cm$$

$$z+\frac{Z}{2}=0+\frac{0.5cm}{2}=0.25cm \quad z-\frac{Z}{2}=0-\frac{0.5cm}{2}=-0.25cm$$

$$C=\frac{1000\frac{mg}{l}}{8}\left(erfc\left(\frac{0}{31.62cm}\right)\right)\left(erf\left(\frac{0.5cm}{14.14cm}\right)-erf\left(\frac{-0.5cm}{14.14cm}\right)\right)\left(erf\left(\frac{0.25cm}{10cm}\right)-erf\left(\frac{-0.25cm}{10cm}\right)\right)$$

$$C=125\frac{mg}{l}(erfc(0))(erf(0.0354)-erf(-0.0354))(erf(0.025)-erf(-0.025))$$

$$C=125\frac{mg}{l}(1)(0.0399-(-0.0399))(0.0282-(-0.0282)) \qquad C=0.56\frac{mg}{l}$$

What do you make of the concentration relative to the C we obtained for the slug source?

How much mass enters the system in 1000sec? M = CQT = CAV$_D$T

How would you go about developing a contour map of the plume?

If you did not know the dispersivities, how could you use this equation to estimate them?

How might you set up the problem if 8g/d arrived at the water table over a 1m^2 area in an aquifer with the properties and conditions used for the example?

What do you make of the concentration relative to the C we obtained for the slug source?

The slug source concentration at 50cm and 1000sec is 2 orders of magnitude higher. This is intuitively reasonable. The slug added 1000mg at time zero.

How much mass enters the system in 1000sec? ? M = CQT = CAV$_D$T

With the continuous source we have 1000mg/l entering an area of 0.5cm^2 for 1000sec at a Darcy velocity of 0.01cm/sec so the mass is lower and mass that entered later has not had the chance to travel far.

M = Conc Area Velocity$_{Darcy}$ Time

$$\text{Total Mass} = \frac{1000mg}{liter} * \frac{1liter}{1000cm^3} * 0.5cm^2 * \frac{0.01cm}{sec} * 1000sec = 5mg$$

On the other hand all the mass for the slug case has been dispersing for the full 1000sec.

How would you go about developing a contour map of the plume?

For a given point in time calculate C at many x,y,z values then map then and contour them

If you did not know the dispersivities, how could you use this equation to estimate them?

Collect concentration data by sampling water in the field then create a contour map using different values of dispersivity until a good match is obtained. There are some automated techniques for calibration that we will not go into here.

How might you set up the problem if 8g/d arrived at the water table over a 1m^2 area in an aquifer with the properties and conditions used for the example?

Use formula for 2D downward spreading with a small Z, perhaps 0.0001 cm, and Y=100cm.
The formula assumes the Q is the area of the source * DarcyV = 0.01cm^2 * 0.01cm/sec
So to get 8g/d Co needs to be = M / (Area Velocity$_{Darcy}$ Time) =

$$\text{Concentration} = 8000mg \Big/ \left(0.01cm^2 * \frac{0.01cm}{sec} * 86400sec\right) = \frac{926mg}{cm^3}\ \frac{1liter}{1000cm^3} = \frac{9.26mg}{liter}$$

For a material with a half-life of 12 yrs, how much is left after 40 yrs? (Hint figure it as a % of initial mass)

$N = N_o e^{(-\lambda t)}$ $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{12\text{yrs}} = 0.05775\text{yrs}^{-1}$$

$$N = N_o e^{(-\lambda t)}$$

$$N = 1 * e^{(-0.05775\text{yrs}^{-1} * 40\text{yrs})} = 0.099$$

or about 10%

It is often said that material is essentially gone after 7 half-lives. How much is left then?

$N = N_o e^{(-\lambda t)}$ $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{1\text{unit}} = 0.693\text{units}^{-1}$$

$$N = N_o e^{(-\lambda t)}$$

$$N = 1 * e^{(-0.693\text{units}^{-1} * 7\text{units})} = 0.0078 \sim 0.008$$

or less than 1%

What is the Retardation Coefficient for a site with
K_d = 0.01 ml/mg
effective porosity of 0.3
particle density of 2.65 g/cc

$$R = \frac{V_{water}}{V_{contaminant}} = \left(1 + \frac{\rho_b}{\phi_e} K_d\right)$$

$$\rho_b = (1-0.3) * 2.65 \frac{g}{cc} \frac{1000mg}{g} \frac{1cc}{1ml} = 1855 \frac{mg}{ml}$$

$$R = \frac{V_{water}}{V_{contaminant}} = \left(1 + \frac{\rho_b}{\phi_e} K_d\right)$$

$$R = \left(1 + \frac{1855 \frac{mg}{ml}}{0.3} 0.01 \frac{ml}{mg}\right) = 62.8 \sim 63$$

What is the Retardation Coefficient for a site with
Ground water velocity = 0.05 cm/sec
Contaminant velocity = 0.0009 cm/sec

$$R = \frac{V_{water}}{V_{contaminant}} = \left(1 + \frac{\rho_b}{\phi_e} K_d\right)$$

$$R = \frac{V_{water}}{V_{contaminant}} = \left(1 + \frac{\rho_b}{\phi_e} K_d\right)$$

$$R = \frac{V_{water}}{V_{contaminant}} = \frac{0.05 \frac{cm}{sec}}{0.0009 \frac{cm}{sec}} = 55.56 \sim 56$$