Physics 300: Fundamental Equations

Special Relativity

Length contraction and Time dilation:	$L = L_0/\gamma(v)$	$T = T_0\gamma(v)$
Lorentz transformations:	$x = \gamma(v)(x' + vt')$	$t = \gamma(v)(t' + vx'/c^2)$
Velocity addition:	$u_x = \frac{u'_x + v}{1 + vu'_x/c^2}$	$u_y = \frac{u'_y}{\gamma(v)(1 + vu'_x/c^2)}$
Doppler shift (approaching):	$f = f_0\sqrt{\frac{1+\beta}{1-\beta}}$	$\lambda = \lambda_0\sqrt{\frac{1-\beta}{1+\beta}}$
Inertial mass and momentum:	$m_{\text{inertial}} = \gamma(u)m$	$\vec{p} = \gamma(u)m\vec{u}$
Total Energy and Kinetic Energy:	$E = \gamma(u)mc^2$	$KE = mc^2(\gamma(u) - 1)$
Four vectors ($\mu = 0, 1, 2, 3$):	$x^\mu = (ct, \vec{x})$	$p^\mu = (E, \vec{p}c)$
Invariants:	$s^2 = -(x^\mu)^2 = x^2 - c^2t^2$	$m^2c^4 = (p^\mu)^2 = E^2 - p^2c^2$

Nuclear Binding Energy: $BE = (ZM[^1H] + NM[n] - M[A])c^2$

Nuclear Reaction Energy ($A + B \to C + D$): $Q = (M[A] + M[B] - M[C] - M[D])c^2$

where $\beta = v/c$ and $\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}}$.

Early Quantum Theory

Blackbody radiation:	$E(\lambda) = \frac{hc}{\lambda}$	$\mathcal{I}(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{E(\lambda)/k_BT} - 1}$
Wein and Stefan-Boltzmann Laws:	$\lambda_M T = 2.898 \times 10^{-3}$ m K	$R = \sigma T^4$
Photoelectric and Compton effects:	$eV_{stop} = hc/\lambda - \phi$	$\Delta\lambda = \lambda_C(1 - \cos\theta)$
Rutherford scattering:	$\frac{d\sigma}{d\Omega} = \frac{D_c^2}{16\sin^4\theta/2}$	$D_c = \frac{Z_\alpha Z_T e^2}{4\pi\epsilon_0 K}$
	$N_{det} = (ntA_T)\frac{N_{inc}}{A_T}\frac{d\sigma}{d\Omega}(\frac{A_{det}}{R^2})$	
Bohr Model:	$R_n = n^2a_0$	$E_n = -\frac{E_0}{n^2},\ \ E_0 = \frac{1}{2}\frac{e^2}{4\pi\epsilon_0 a_0}$
	$a_0 = \frac{4\pi\epsilon_0\hbar^2}{e^2 m_{\text{red}}}$	$m_{\text{red}} = \frac{m_1m_2}{m_1+m_2}$
Moseley's x-ray formulas:	$E_{\gamma_K} = (Z-1)^2E_0(1 - n^{-2})$	$E_{\gamma_L} = (Z - 7.4)^2E_0(\frac{1}{2^2} - n^{-2})$
de Broglie and Bragg diffraction:	$p = h/\lambda = \hbar k$	$2d\sin\theta = n\lambda$

Schroedinger Theory

Wave Parameters:	$\omega = 2\pi/T = 2\pi f$	$k = 2\pi/\lambda$
Heisenberg uncertainty relations:	$\Delta x\Delta p_x \geq \hbar/2$	$\Delta t\Delta E \geq \hbar/2$

Time-dep. Schroedinger equation: $\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$

Static Schroedinger equation: $\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi(x) = E\psi(x)$, where $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$

Boundary conditions: $\Psi(x,t)$ and $\frac{\partial\Psi(x,t)}{\partial x}$ continuous at each boundary.

Square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$ $\qquad E_n = \frac{\hbar^2n^2\pi^2}{2mL^2}$

Operators: $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ $\qquad \hat{x} = x$ $\qquad \hat{E} = i\hbar\frac{\partial}{\partial t}$

Probability and expectation values: $dP(x \to x + dx) = \psi^*(x)\psi(x)dx$ $\qquad < O >= \int dx\ \psi^*(x)\hat{O}\psi(x)$

Harmonic oscillator: $\psi_n(x) = N_nH_n(x/\alpha)e^{-\frac{1}{2}(x/\alpha)^2}$ where $\alpha = \sqrt{\hbar/(m\omega)}$

$E_n = \hbar\omega(n + 1/2)$ and $\omega = \sqrt{k_{sp}/m}$

and N_n normalizes the wavefunction: $1 = \int_{-\infty}^{+\infty}|\psi_n(x)|^2dx$

Atoms and Molecules

Radial Schroedinger Eq. :	$\left(-\frac{\hbar^2}{2m_e}(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r})+\frac{\hbar^2 l(l+1)}{2m_e r^2}+V(r)\right)R_{nl}(r)=E_{nl}R_{nl}(r)$
Hydrogen wavefunctions:	$\psi_{nlm_l}(r,\theta,\phi)=R_{nl}(r)Y_{lm_l}(\theta,\phi)$ $\quad R_{10}(r)=\sqrt{\frac{2}{a_0^3}}\,e^{-r/a_0}$ (g.s.)
Spherical Harmonics:	$\hat{L}^2 Y_{lm_l}(\theta,\phi)=\hbar^2 l(l+1)Y_{lm_l}(\theta,\phi)$ $\quad \hat{L}_z Y_{lm_l}(\theta,\phi)=\hbar m_l Y_{lm_l}(\theta,\phi)$
Hydrogen quantum numbers:	$n=1,2,3,\ldots;\ n>l;\ -l\le m_l\le +l;$ multiplicity $=2l+1$
Electron magnetic moments:	$\mu_z=-g_j\mu_B m_j$, where for orbital a.m., $g_l=1$ and for spin a.m., $g_s=2$
Zeeman and Stern-Gerlach:	$U=-\vec{\mu}\cdot\vec{B}=g_j\mu_B m_j B$ $\quad F_z=g_j\mu_B m_j\frac{dB}{dz}$
Angular momentum addition:	$\vec{J}_{tot}=\vec{J}_1+\vec{J}_2$ $\quad |j_2-j_1|\le j_{tot}\le j_1+j_2$
(similarly for $\vec{L}$ and $\vec{S}$)	$(2j_1+1)(2j_2+1)=\sum_i(2j_i+1)$, where j_i are allowed values of j_{tot}
Spectroscopic notation:	$\{s,p,d,f,g,h,\ldots\}\Longrightarrow l=\{0,1,2,3,4,5,\ldots\}$
Diatomic molecules:	$E_{rot}=\frac{L^2}{2I}=\frac{\hbar^2 l(l+1)}{2m_{red}R^2};\ E_{vib}=\hbar\omega(n+\frac{1}{2})$ where $\omega=\sqrt{\frac{k_{sp}}{m_{red}}}$
	$k_{sp}=V''(r_0)$ where r_0 is the separation; transitions: $\Delta l=\pm 1$

Solid State

Crystal energetics:	$U(r)=-\alpha\frac{e^2}{4\pi\epsilon_0 r}+\frac{A}{r^n}$	Equilibrium: $\frac{dU}{dr}=0$
Fermi gas model:	$d^3N=2\frac{Vol\,d^3p}{h^3}$	$n=N/Vol=\frac{8\pi p_F^3}{3h^3}$
Fermi-Dirac Distribution:	$f_{FD}(E,T)=(e^{(E-E_F)/kT}+1)^{-1}$	$E_F=\frac{p_F^2}{2m}$
Drude model of conduction:	$\rho=\frac{m_e\bar{v}}{e^2 n_e l_{MFP}}$	$l_{MFP}=\frac{1}{n\sigma_{scatt}}$

Nuclear

Nuclear size and shape:	$R=r_0A^{1/3}$ $\quad \rho(r)=\rho_0(1+e^{(r-R)/a})^{-1}$
Nuclear Energetics:	$BE[^AZ]=\left(ZM[^1H]+NM[n]-M[^AZ]\right)c^2$
Semiempirical Mass Formula:	$BE_{SE}[^AZ]=a_VA-a_AA^{2/3}-a_{sym}(N-Z)^2A^{-1}-a_cZ(Z-1)A^{-1/3}$
Decay law:	$N(t)=N_0e^{-t/\tau}$, $\frac{dN(t)}{dt}=-N(t)/\tau$ $\quad$ Half-life: $t_{1/2}=ln(2)\tau$
Nuclear radiation::	α-decay: $^AZ\rightarrow\ ^{(A-4)}(Z-2)+^4He$
	β^--decay: $^AZ\rightarrow\ ^A(Z+1)+e^-+\bar{\nu}_e$
	γ-decay: $^AZ^*\rightarrow^A Z+\gamma$

Constants

$c=3.00\times10^8$ m/s	$m_e=.511$ MeV/c^2	$m_p=938.27$ MeV/c^2	$m_n=939.57$ MeV/c^2
$m_d=1875.6$ MeV/c^2	1 eV $=1.6\times10^{-19}$ J	u (amu) $=931.49$ MeV/c^2	$h=6.63\times10^{-34}$ J-s
$hc=1240$ eV-nm	$\hbar c=197.3$ eV-nm	$k_B=1.38\times10^{-23}$ J K^{-1} $=8.617\times10^{-5}$ eV K^{-1}	
$\sigma=5.67\times10^{-8}$ W m^{-2} K^{-4}	$\frac{e^2}{4\pi\epsilon_0}=1.44$ eV-nm	For Hydrogen:	$a_0=0.0529$ nm, $E_0=13.6$ eV

$\lambda_C=\frac{hc}{m_ec^2}=2.43\times10^{-3}$ nm

$\mu_B=\frac{e\hbar}{2m_e}=5.7884\times10^{-5}$ eV/T

Madelung constant for cubic crystal: $\alpha=1.7476$

Nuclear shape parameters: $r_0=1.2$ fm, $\rho_0=0.14$ fm^{-3}, $a=0.65$ fm

Semiempirical mass formula parameters: $\{a_V=14$ MeV, $a_A=13$ MeV, $a_{sym}=19$ MeV, $a_c=.72$ MeV$\}$

Carbon-14 dating parameters: $\tau=8267$ yr, atmospheric ^{14}C/^{12}C $=1.2\times10^{-12}$