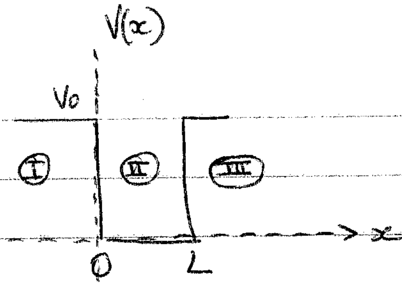
Finite Square-Well potential



$$V(x) = \begin{cases} V_0 & x \leq 0 \\ 0 & x < x < L \\ V_0 & x \geq L \end{cases}$$

Looking at bound states ($E < V_0$)

Region I and III
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} = (E - V_0)$$

Posing: $\alpha^2 = 2m(V_0 - E)/\hbar^2$
$$\frac{d^2\psi}{dx^2} = \alpha^2\psi$$

Solutions: $e^{\alpha x}$ and $e^{-\alpha x}$; Positive exponential have to be rejected [not physical]

e.g. the exponential has to die out when $x \to \mp\infty$ [but it is not strictly 0]

Region I: $\psi_I(x) = Ae^{\alpha x}$ $(x \leq 0)$

Region III: $\psi_{III}(x) = Be^{-\alpha x}$ $(x \geq L)$

Solution for region II known from infinite square-well potential:

$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx} \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

Boundary Conditions: $x = 0$ $\quad \psi_I(x) = \psi_{II}(x)$

$$\frac{d}{dx}\psi_I(x) = \frac{d}{dx}\psi_{II}(x)$$

$x = L$ $\quad \psi_{II}(x) = \psi_{III}(x)$

$$\frac{d}{dx}\psi_{II}(x) = \frac{d}{dx}\psi_{III}(x)$$