Uncertainty Principle:

EXAMPLE 1: $\Delta p \Delta x \geqslant \frac{\hbar}{2}$

$$\Rightarrow \Delta p \sim \frac{\hbar}{2}\frac{1}{\Delta x} \quad \Rightarrow p_{min} \sim \frac{\hbar}{2}\frac{1}{\Delta x}$$

Kinetic Energy [Classical]: $K_{min} = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2 = \frac{1}{2m}\Delta p^2$

$$\hookrightarrow K_{min} = \frac{1}{2m} p_{min}^2$$

Introducing c^2: $K_{min} = \frac{1}{2mc^2}(pc)^2$

$$\hookrightarrow K_{min} = \frac{1}{2mc^2}\left(\frac{\hbar c}{2\Delta x}\right)^2 = \frac{(\hbar c)^2}{8mc^2\,\Delta x^2}$$

with: $\hbar c = 3.1615 \times 10^{-26}$ J.m
$= 197.33$ eV. nm

$\Delta x = 16 \times 10^{-15}$ m $= 16 \times 10^{-6}$ nm

$mc^2 = 938.27$ MeV $= 938.27 \times 10^6$ eV

$$K_{min} = \frac{(197.33)^2\ [\text{eV.nm}]^2}{8\,[938.27\times10^6)(16\times10^{-6})^2\,[\text{eV.nm}^2]} = 20.26 \text{ keV}$$

Uncertainty Principle:

EXAMPLE 2:

(a) Q-value: $Q = m_p c^2 - m_n c^2 - m_\pi c^2$

$= 938.8 - 939.6 - 139.6$ MeV

$= -140.4$ MeV

Energy "borrowed": $\Delta E = 140.4$ MeV

(b) Uncertainty principle: $\Delta E \, \Delta t \geqslant \frac{\hbar}{2} \Rightarrow \Delta t \sim \frac{\hbar}{2} \frac{1}{\Delta E}$

with: $\hbar = 6.5821 \times 10^{-16}$ eV.s

$$\rightarrow \Delta t \sim \frac{6.5821 \times 10^{-16} \, [\text{eV. s}]}{2 \times (140.4 \times 10^6) \, [\text{eV}]} = 2.34 \times 10^{-24} \text{ s.}$$

(c) $d = c\Delta t = 3.0 \times 10^8 \, \text{m/s} \times 2.34 \times 10^{-24} \, \text{s} = 0.7 \times 10^{-15} \, \text{m} = 0.7$ fm.

[order of magnitude of the strong interaction]