

PHGN511 Homework #5
Due Friday, Oct. 1, 2004 at the beginning of class

Show your work

1. Evaluate :

$$I = \int_{-1}^1 \frac{dx}{\sqrt{(1-x^2)(1+x^2)}}$$

Hint: This one is a bit tricky. It's all about picking the right path. Notice the limits on the integral and think about the branch cut location we got for $\sqrt{1-x^2}$. This should give you a clue about what contour might help.

2. Find the general solution of the ordinary differential equation:

$$x^2 y' + y^2 = xyy'$$

Hint: This kind of first order differential equation is said to be homogeneous. If you write it in the form $A(x,y)dx + B(x,y)dy=0$, then A and B turn out to be homogeneous functions of the same degree. This is a bit of a formal definition. The way I think about it is that units match. In the above, if x and y are in meters, you can see that units match across the equation. You have the same overall powers of x and y taken together in each term. A change of variables that usually makes such an equation separable is $y = vx$. Notice that this is a change to a unitless variable v. This kind of unitless transformation is often useful. Try this and solve the equation. In this case its going to be easier to get x in terms of y then vice versa.

3. In an electrical circuit with a capacitor C in series with resistance R and time varying voltage:

$$V = V_0 \left(\frac{t^2}{\tau^2} \right) e^{-\frac{t}{\tau}}$$

the charge, q, on the capacitor obeys:

$$R \frac{dq(t)}{dt} + \frac{1}{C} q(t) = V.$$

Find q(t) if q(0)=0.

4. Butkov 3.4