

PHGN511 Homework #6

Due Friday, Oct. 8, 2004 at the beginning of class

Show your work and with each Frobenius type problem try to argue (before beginning) what kind of a solution you expect, e.g. power series, Laurent etc., based on the form of the D.E.

1. Butkov 3.3
2. Butkov 3.7 Be very careful with the recursion relation on this one. Depending upon how you solve the problem, you may end canceling two terms which turn out, for a particular value of n, to give you zero/zero.
3. Butkov 3.11
4. Butkov 3.14
5. A quantum mechanical analysis of the Stark shift (change in energy levels due to an applied electric field) leads to the differential equation:

$$\frac{d}{dx} \left(x \frac{du}{dx} \right) + \left(\frac{1}{2} E x + \alpha - \frac{m^2}{4x} - \frac{1}{4} F x^2 \right) u = 0$$

where α is a separation of variables constant, E is the total energy, and F is a constant proportional to the applied electric field.

- a) Discuss the behavior of the differential equation. Are there singular points? Do you expect to find a Frobenius solution. Explain why.
- b) Obtain the indicial equation in a Frobenius type solution.
- c) Using the larger root, develop a series solution valid about $x=0$. Generate enough terms so that you have included the leading order effect due to the perturbing electric field.