PHGN511 Homework #11
Due Friday, Nov. 19, 2004 at the beginning of class
Show your work.

1. a) Using recursion relations a) and b) on Butkov page 346, show

$$p'_{l+1} - p'_{l-1} = (2l+1)p_l .$$

For what values of *l* does this expression apply?

b) A spherical surface of radius R has charge uniformly distributed over its surface with density $Q/4\pi R^2$ except for a spherical cap at the north pole defined by the cone Cos $\theta = \alpha$. Show that the potential inside the spherical shell is given by:

$$\Phi = \frac{Q}{2}\sum_{l=0}^{\infty}\frac{1}{2l+1}\left[P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)\right]\frac{r^l}{R^{l+1}}P_l\cos(\theta)$$

This problem is a standard graduate level E&M problem (it's not the first one you have done in this class!). It may take you a bit to do so start early. You may find part a) useful. Also, this problem involves two regions (inside the sphere and outside the sphere) that obey Laplace's equation. To solve it you must match boundary conditions at the sphere surface where the charge exists and where Laplace's equation doesn't apply. Most of you should know those matching conditions. They come from Maxwell's equations. Since this is a math physics course, however, I will give them to you. The potential needs to be continuous across the sphere, but the radial derivative of the potential (the radial component of the gradient) will be discontinuous. In particular $\frac{\partial\Phi}{\partial r}$ will exhibit a step-like discontinuity of $-4\pi\sigma$ as r increases across the sphere where σ is the local surface charge density. Note this problem is in Gaussian units.

2. Butkov 9.6 As part of this problem, also show $U_0(x)=1$ and, from the recursion relation, find the next 3 polynomials. Now use Mathematica to expand the generating function and show that the polynomials you get from the expansion are in agreement with those from the recursion relation.

3. A conducting sphere of radius, a, is placed in an electric field, E, pointing along the z-axis. Because the sphere is conducting, it is equipotential. We will call its potential the constant, φ_s . Because the electric field is present, we expect the potential for r>>1 to be of the form $\varphi = C - E z = C - E r \cos\theta$, where C is a constant (this expression gives us a type of boundary condition which is different from what we have seen before, the behavior of the function instead of a specific value). Now, find an expression for φ which is valid for r > a in terms of Legendre polynomials. Simplify it as much as possible (its pretty simple in the end).