Final Exam Phgn511 Math Physics
December 9, 2003
Show your work on all problems

1. (10 pts) A function f(x) is expanded in a Legendre series

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x).$$

Show that

$$\int_{-1}^{1} [f(x)]^2 dx = \sum_{n=0}^{\infty} \frac{2a_n^2}{2n+1}.$$

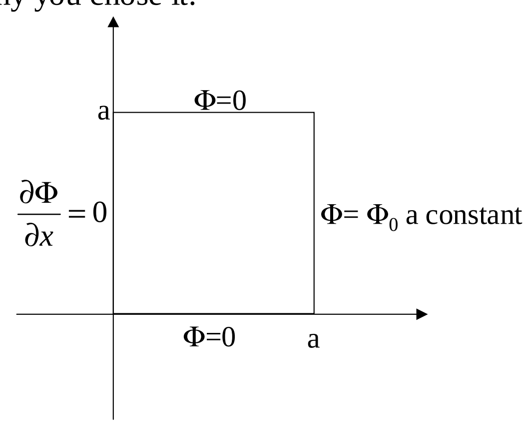
As you do this proof, certain steps are going to require assumptions about the sum that gives you f(x). Be sure to point these out.

2. (10pts) Consider the following integral:

$$I = \int_0^{\infty} \frac{\sqrt{x}}{1+x^4} dx$$

What contour would you choose in the complex plane to evaluate it? No need to find the integral, just give a contour and briefly explain why you chose it.

3. (20 pts) We want to find the electrostatic potential inside a 2-dimensional box of side length a. There is no charge inside the box so the potential satisfies Laplace's equation in 2-d, $\nabla^2\Phi = 0$. The boundary conditions are as given in the figure. Using separation of variables, find the potential everywhere inside the box.



4. (20pts) We want to find the Green's function for the differential equation

$\frac{\partial^2\Phi}{\partial x^2} - k_0^2\Phi = f(x)$ with the boundary conditions $\Phi(\pm\infty) = 0$.

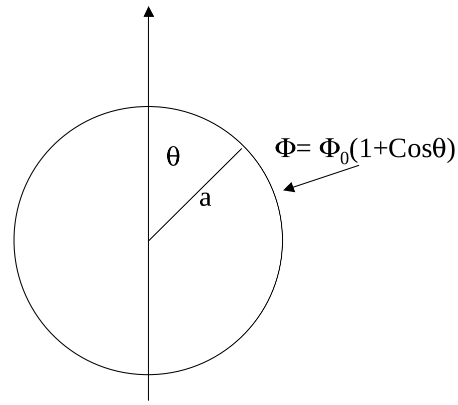
a) Obtain the Green's function by direct construction.

b) Now, using a Fourier transform approach, show that

$$G(x,x') = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{ikx'}e^{-ikx}}{k_0^2 + k^2} dk$$

c) Evaluate this integral and show you obtain the same result as in part a.

5. (20 pts) The electrostatic potential on a spherical surface of radius a is given by Φ_0 (1+Cosθ). We will assume there is no charge anywhere (except possibly on the surface), so the potential satisfies Laplace's equation $\nabla^2\Phi = 0$ for the regions r < a and r > a and the potential is bounded in both regions. Find expressions for the potential for both r > a and r < a. You don't actually have to go through the whole process of separating the variables, since you should know the form of the solution, but, be sure to explain your approach. For example, if you drop some terms, briefly explain why you aren't including them.



6. (20 pts) The amplitude of a circular membrane of radius, a, with its edges clamped obeys the 2-dimensional wave equation $\nabla^2 U = \frac{1}{\mathrm{v}^2}\frac{\partial^2 U}{\partial t^2}$ and satisfies Dirichlet boundary conditions. Perform the separation of variables to obtain a solution to this problem (what I am looking for is the form of the eigenfunction from which a general solution would be built). Be careful to explain terms you discard, and also comment specifically on how the separation constants are set, or values that they must have due to the symmetry of the problem or the boundary conditions etc.