

Midterm Phgn511 Math Physics

Oct. 10, 2003

All problems of equal value

Show your work on all problems

1) Locate and classify the isolated singularities (e.g. a simple pole) of the following three functions:

$$\frac{z^2}{1+z}, \quad \frac{1-\cos(z)}{z}, \quad ze^{\frac{1}{z}}.$$

In each case give a brief justification for your answer.

2) Consider the homogenous differential equation:

$$x^2 y'' + (x^2 + x)y' - y = 0.$$

a) Discuss where you expect singular points in the solution

b) Obtain two linearly independent solutions to the differential equation using the Frobenius method. You can just list several terms in each series, and the recursion relation that gives them.

3) Find the Fourier transform of $f(x) = \frac{1}{\cosh(x)}$.

4) Consider $f(z) = \frac{1}{(1-z)(2-z)}$

a) Find a Laurent or Taylor series expansion for $f(z)$ about the point $z = 0$, which is valid when $|z| \gg 1$. What is the domain of convergence of the series?

b) Discuss any other series expansions of this type which exist for the function and their domain of validity in the complex plane. There is no need to find them, just describe what types of series you expect, and the domain in the complex plane for which each series is expected to converge.