Physics 520 – Graduate Quantum Mechanics
Homework I, due Wednesday, January 16, 2008 at 8:30 a.m.

1. **Reading**

   G&Y Sections 3.7(c)-(d), 2.6, and 2.7 (in that order)

2. **Time-Dependent Perturbation Theory for the Qubit**

   Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows: $V_{11} = V_{22} = 0$, $V_{12} = \gamma e^{i\omega t}$, $V_{21} = V_{12}^*$; $\gamma \in \mathcal{R}$. At $t = 0$ it is known that only the lower level is populated – that is, $c_1(0) = 1$, $c_2(0) = 0$.

   (a) Sketch our treatment from last semester of the exact solution of this problem. Find $|c_1(t)|^2$ and $|c_2(t)|^2$.

   (b) Do the same problem using time-dependent perturbation theory to lowest nonvanishing order. Compare the two approaches for small values of γ. Treat the following two cases separately:

   (i) ω very different from $\omega_{12} \equiv (E_2 - E_1)/\hbar$;

   (ii) $\omega \simeq \omega_{12}$.

3. **Time-Dependent Perturbation Theory for the Hydrogen Atom**

   Consider a hydrogen atom in its ground state, which, beyond time $t = 0$, is subject to a spatially uniform time-dependent electric field $\mathcal{E}_0 e^{-t/\tau}$. Treating the electric field as a perturbation, calculate to first order the probability of finding the atom in the first excited state $(n = 2, \ell = 1, m)$.

4. **Time-Dependent Perturbation Theory for a Charged Particle in a Box**

   A particle of mass m and charge q moves in one dimension between the impenetrable walls of an infinite square-well potential of length L centered around the origin.

   (a) Consider a weak uniform electric field of strength $\mathcal{E}$ that acts on the particle. Calculate the first non-trivial correction to the particle's ground-state energy. What is the probability of finding the particle in the first excited state?

   (b) Consider now the case of a time-dependent electric field of the form $\mathcal{E}(t) = \mathcal{E}_0 \theta(t) e^{-t/\tau}$. Calculate the transition probability from the ground state of the system to the first excited state to first order in time-dependent perturbation theory for late times $t \gg \tau$.

   Hint: You may find useful the following sum.

$$\sum_{n=1}^{\infty} \left[\frac{1}{(4n^2-1)^3} + \frac{4}{(4n^2-1)^4} + \frac{4}{(4n^2-1)^5} \right] = \frac{1}{2} - \frac{\pi^2}{64} \left(\frac{7}{4} + \frac{\pi^2}{12} \right) \qquad (1)$$