Physics 521 – Graduate Quantum Mechanics
Homework XI, due Wednesday, April 23, 2008 at 8:30 a.m.

This homework has two pages.

1. **Finite Spherical Well and the Born Approximation**

   Determine, in the first order Born approximation, the total cross section for a finite spherical well of depth $-V_0 < 0$ and radius a. Find the limiting expressions for $ka \ll 1$ and $ka \gg 1$, where $\hbar k$ is the momentum of the incident wave.

2. **Yukawa Potential and the Born Approximation**

   Work through the Yukawa potential scattering problem as described in the notes.

   (a) Plot the Yukawa potential. Physically motivate its form in terms of screening. In what limit would the Coulomb potential be reproduced? What is the meaning of the interaction mass μ?

   (b) Derive the form of the differential and total cross sections in the first order Born approximation.

   (c) Determine the differential and total cross sections in the second order Born approximation.

   (d) Develop a criterion for when the first order approximation is sufficient.

   (i) What is the criterion for low energies? Physically interpret your result. *Hint: A bound state of the Yukawa potential occurs for* $2mZZ'e^2a^2/\hbar^2 \geq 2.7$.

   (ii) What is the criterion for high energies? Again, physically interpret your result.

   (iii) Show explicitly that, when your criteria of parts (c)(i) and (c)(ii) hold, the second order contribution is much smaller than the first.

3. **Scattering Resonances of a Delta-Shell Potential**

   Consider an attractive delta-shell potential

   $$V(r) = -\frac{\hbar^2 \lambda}{2\mu}\delta(r-a)\,, \tag{1}$$

   where $\lambda > 0$.

   (a) Calculate the phase shift $\delta_l(k)$, where l is the angular momentum quantum number.

   (b) In the case $l = 0$, investigate the existence of bound states by examining the analytic properties of the partial scattering amplitude. Are there resonances?

4. **Radial Lippman-Schwinger Equation**

   The radial Green's function is defined by the equation

   $$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{l(l+1)}{r^2} + k^2\right]\mathcal{G}_{k,l}(r,r') = \frac{1}{r^2}\delta(r-r') \tag{2}$$

   (a) Verify the choice

   $$\mathcal{G}^{(-)}_{k,l}(r,r') = C\left[\theta(r'-r)j_l(kr)h_l^{(-)}(kr') + \theta(r-r')j_l(kr')h_l^{(-)}(kr)\right] \tag{3}$$

by substituting Eq. (3) into Eq. (2). Also find the normalization constant C.

(b) Show that the radial wave function satisfies the integral equation

$$R_{k,l}(r) = j_l(kr) + \int_0^\infty dr' r'^2 \frac{2m}{\hbar^2} V(r') \mathcal{G}_{k,l}(r, r') R_{k,l}(r') \,. \tag{4}$$