Physics 521 – Graduate Quantum Mechanics
Homework II, due Wednesday, January 23, 2008 at 8:30 a.m.

1. **Reading**

   G&Y Section 4.2

2. **Momentum space propagator**

   Derive an explicit expression for $\langle \mathbf{p}', t' | \mathbf{p}, t \rangle$.

3. **Equivalence of propagator and wave mechanics**

   Prove that the general propagator satisfies the time-dependent Schrodinger equation

4. **Imaginary time propagator**

   Define the partition function as

$$Z = \int d^3x' K(\mathbf{x}', t; \mathbf{x}', 0)|_{\beta = it/\hbar} \,. \tag{1}$$

   Show that the ground state energy E_0 is given by

$$E_0 = \lim_{\beta \to \infty} -\frac{1}{Z} \frac{\partial Z}{\partial \beta} \,. \tag{2}$$

   Illustrate this for a particle in a one-dimensional box.

   Hint: You may find Sakurai section 2.5 useful.

5. **Propagator symmetries**

   Consider the propagator defined as

$$K(\mathbf{x}, \mathbf{x}', t - t') = \langle \mathbf{x} | e^{-i(t-t')\hat{H}/\hbar} | \mathbf{x}' \rangle \,, \tag{3}$$

   i.e., $\hat{H} \neq \hat{H}(t)$.

   (a) Show that when the system, i.e., the Hamiltonian, is invariant under spatial translations then the propagator has the property

$$K(\mathbf{x}, \mathbf{x}', t - t') = K(\mathbf{x} - \mathbf{x}', t - t') \,. \tag{4}$$

   (b) Show that when the energy eigenfunctions are real, i.e., $\psi_E(\mathbf{x}) = \psi_E^*(\mathbf{x})$, the propagator has the property

$$K(\mathbf{x}, \mathbf{x}', t - t') = K(\mathbf{x}', \mathbf{x}, t - t') \,. \tag{5}$$

   (c) Show that when the energy eigenfunctions are also parity eigenfunctions, the propagator has the property

$$K(\mathbf{x}, \mathbf{x}', t - t') = K(-\mathbf{x}, -\mathbf{x}', t - t') \,. \tag{6}$$

   (d) Show that the propagator always has the property

$$K(\mathbf{x}, \mathbf{x}', t - t') = K^*(\mathbf{x}', \mathbf{x}, t' - t) \,. \tag{7}$$