Physics 521 – Graduate Quantum Mechanics
Homework IX, due Wednesday, March 26, 2008 at 8:30 a.m.

1. **Reading**

   G&Y Section 4.4 and 8.1 - 8.3

2. **Time-reversal in the Momentum Space Functional Representation**

   Let $\phi(\vec{p})$ be the momentum space functional representation of state $|\alpha\rangle$. Is the momentum space representation for the time reversed state $\hat{\Theta}|\alpha\rangle$ given by $\phi(\vec{p})$, $\phi(-\vec{p})$, $\phi^*(\vec{p})$, or $\phi^*(-\vec{p})$? Justify your answer.

3. **Time-reversal in the Angular Momentum Ket Representation**

   (a) What is the time-reversed state corresponding to $\hat{\mathcal{D}}(R)|j,m\rangle$?

   (b) Using the properties of time reversal and rotations, prove

   $$\mathcal{D}^{(j)}_{mm'}(R) = (-1)^{m-m'}\mathcal{D}^{(j)}_{-m',-m}(R)\,. \tag{1}$$

   (c) Prove $\hat{\Theta}|j,m\rangle = i^{2m}|j,-m\rangle$.

4. **Scattering of a wavepacket by a potential barrier/well in 1D**

   Numerically solve the 1D scattering problem for a wavepacket on a finite square barrier/well of form $V(x) = V_0$ for $|x| \le a/2$, $V(x) = 0$ otherwise. Take as your initial state a right-traveling, normalized Gaussian wavepacket far to the left of the barrier/well, i.e.,

   $$\psi(x, t=0) = \pi^{-1/4}x_0^{-1/2}\exp(-(x+L)^2/2x_0^2)\exp(imvx/\hbar), \tag{2}$$

   where x_0 is the width, v is the velocity of the right-traveling wavepacket, and $-L$ is the initial center of the wavepacket with $L \gg a/2$. *Your solution to the following must be supplemented via an attachment to a single e-mail containing your code in Mathematica, Matlab and/or C/C++. No Java, please. You can send one copy of your code for a team of two people, but larger teams will not be accepted; the basic algorithms can be the same as that of another team, but the code must be otherwise unique. The code must be clearly and thoroughly commented. It must compile and run with little or no effort on the grader's part. Specifically, it must compile easily on a standard Linux platform in the physics department and run in less than 15 minutes on a 2GHz processor with 512 MB memory. Mathematica, Fortran, or C/C++ are all ok. If you use Fortran or C/C++ provide a makefile and a readme file to explain how to plot the output with standard Linux software such as gnuplot. Make sure to test it on one of the Linux machines in the graduate office on the second floor!*

   (a) What happens when the width of the Gaussian is much smaller than that of the barrier?

   (b) What happens when the width is much greater than that of the barrier?

   (c) Experiment with your code to determine how the incident velocity v and the "area" of the barrier $mV_0a^2/\hbar^2$ affect the scattering.

   Hint: In order to deal with reflections from the endpoints of your spatial grid, you can either (*i*) choose a long grid and run your code for sufficiently short times, or (*ii*) implement an absorbing barrier by adding an imaginary part to the potential near the endpoints of the grid.