TOOLS FOR SOLVING BIOFILM PROBLEMS

The equations shown here are those referenced in the solutions that follow.

- **Definition of non-dimensional parameters**

$$S^*_{min} = \frac{b'}{Y \cdot \hat{q} - b'} = \frac{b + b_{det}}{Y \cdot \hat{q} - (b + b_{det})} \quad (1)$$

$$S^* = \frac{S}{K} \quad (2)$$

$$K^* = \frac{D}{L} \cdot \sqrt{\frac{K}{\hat{q} \cdot X_f \cdot D_f}} \quad (3)$$

- **Steps for using the steady-state-biofilm model**

a. Calculate S^*_{min}, S^*, and K^* from equations 1, 2, and 3, respectively.

b. Compute $\alpha = 1.5557 - 0.4117 \cdot \tanh(\log_{10} S^*_{min})$ (4)

$\beta = 0.5035 - 0.0257 \cdot \tanh(\log_{10} S^*_{min})$ (5)

c. Solve for S^*_S (implicit equation) $S^*_S = S^* - \frac{1}{K^*} \tanh\left(\alpha \cdot \left(\frac{S^*_S}{S^*_{min}} - 1\right)^{\beta}\right) \cdot \sqrt{2 \cdot \left[S^*_S - \ln(1 + S^*_S)\right]}$ (6)

d. Calculate $J^* = K^* \cdot (S^* - S^*_S)$ (7)

e. Calculate $J = J^* \cdot \sqrt{K \cdot \hat{q} \cdot X_f \cdot D_f}$ (8)

f. Calculate $X_f \cdot L_f = \frac{Y \cdot J}{b'} = \frac{Y \cdot J}{b + b_{det}}$ (9)

g. Calculate $L_f = \frac{X_f \cdot L_f}{X_f}$ (10)

- **Normalized loading curves parameters**

$$S_{min} = K \cdot \frac{b'}{Y \cdot \hat{q} - b'} = K \cdot \frac{b + b_{det}}{Y \cdot \hat{q} - (b + b_{det})} \quad (11)$$

$$S^*_{min} = \frac{S_{min}}{K} = \frac{b + b_{det}}{Y \cdot \hat{q} - (b + b_{det})} \quad (1)$$

$$K^* = \frac{D}{L} \cdot \sqrt{\frac{K}{\hat{q} \cdot X_f \cdot D_f}} \quad (3)$$

J^*_R : use the value of S^*_{min} to determine it from the Cannon curve

$$J_R = J^*_R \cdot \sqrt{K \cdot \hat{q} \cdot X_f \cdot D_f} \quad (12)$$

- **Miscelaneous**

$b' = b + b_{det}$ (13) Steady-state mass balance: $Q \cdot (S^0 - S) = J \cdot (a \cdot V)$ (14) $\theta_X = \frac{1}{b_{det}}$ (15)

- **Steps for using the nonsteady-state-biofilm model**

a. Compute

$$S^* = \frac{S}{K} \quad (2) \qquad \tau = \sqrt{\frac{K \cdot D_f}{\hat{q} \cdot X_f}} \quad (16) \qquad L^* = \frac{L}{\tau} \quad (17) \qquad L_f^* = \frac{L_f}{\tau} \quad (18) \qquad D_f^* = \frac{D_f}{D} \quad (19)$$

b. Estimate a starter value for the effectiveness factor η. If the biofilm is very shallow η approaches to 1. For deep biofilms Eq. 20 can be used. Rittmann and McCarty (1981) suggested Eq. 21.

$$\eta = \sqrt{\frac{D_f \cdot (K + 2 \cdot S_S)}{\hat{q} \cdot X_f}} = \frac{\sqrt{1 + 2 \cdot S^*}}{L_f^*} \quad (20) \qquad \eta = \frac{\tanh(L_f^*)}{L_f^*} \quad (21)$$

c. Compute $S_S^* = 0.5 \cdot \left[(S^* - 1 - L^* \cdot L_f^* \cdot D_f^* \cdot \eta) + \sqrt{(S^* - 1 - L^* \cdot L_f^* \cdot D_f^* \cdot \eta)^2 + 4 \cdot S^*} \right]$ (22)

d. Compute $J^* = L_f^* \cdot D_f^* \cdot \eta \cdot \frac{S_S^*}{1 + S_S^*}$ (23)

e. Compute $S_S^{*\prime} = S^* - J^* \cdot L^*$ (24)

f. Compute $\phi = \frac{L_f^*}{\sqrt{1 + 2 \cdot S_S^{*\prime}}}$ (25)

g. For $\phi \le 1$, compute $\eta' = 1 - \frac{\tanh(L_f^*)}{L_f^*} \cdot \left(\frac{\phi}{\tanh(\phi)} - 1 \right)$ (26)

For $\phi \ge 1$, compute $\eta' = \frac{1}{\phi} - \frac{\tanh(L_f^*)}{L_f^*} \cdot \left(\frac{1}{\tanh(\phi)} - 1 \right)$ (27)

h. If η' is close enough to η go to step i. Otherwise, use η' as the estimator of η, and go to step c.

i. Calculate $J^* = L_f^* \cdot D_f^* \cdot \eta \cdot \frac{S_S^{*\prime}}{1 + S_S^{*\prime}}$ (28)

k. Calculate $J = J^* \cdot \frac{K \cdot D}{\tau}$ (29)