-7-08 Class notes

Summary of components and rate functions:

$$r_{net} = Y \frac{\hat{q} S}{K+S} X_a - b X_a$$

$$r_{ut} = -\frac{\hat{q} S}{K+S} X_a \quad \text{on substrate}$$

$$r_{inert} = (1-f_d) b X_a$$

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new components UAP : BAP ← biomass associated products
SMPs ← utilization associated products

formation

$$r_{UAP} = -k_1 r_{ut}$$

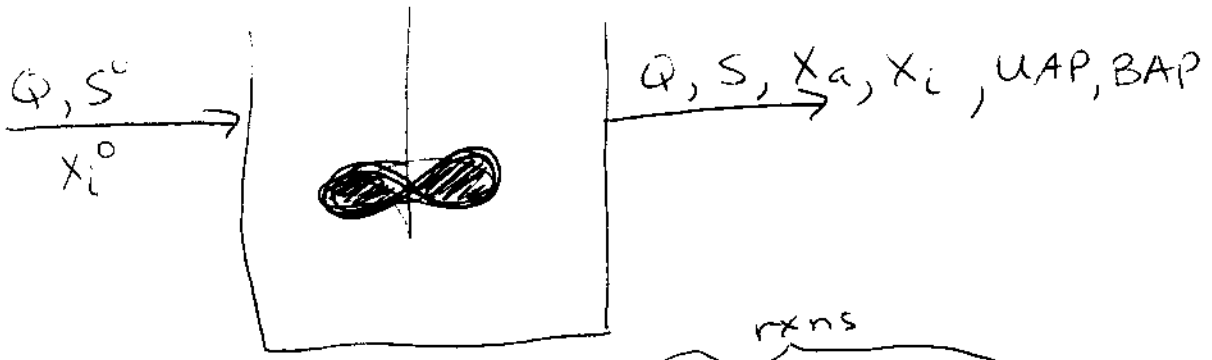
$$r_{BAP} = +k_2 X_a$$

degradation

$$r_{deg-UAP} = -\frac{\hat{q}_{UAP} UAP}{K_{UAP} + UAP} X_a$$

$$r_{deg-BAP} = -\frac{\hat{q}_{BAP} BAP}{K_{BAP} + BAP} X_a$$

Equations for S, X_a, X_i, UAP, BAP for steady state chemostat



$$r_{net} = \frac{Y\hat{q}S}{K+S}X_a - bX_a$$

$$r_{ut} = -\frac{\hat{q}S}{K+S}X_a$$

$$r_{inert} = (1-f_d)bX_a$$

0 = in − out + (formation − consumption) rxns

mass balance

Component	mass balance		equation	
X_a	$0 = 0 - QX_a + \left[\frac{Y\hat{q}S}{K+S}X_a - bX_a\right]V$ (r_{net})	solve for X_a →	$X_a = Y\left(\frac{S^0 - S}{1+b\theta_x}\right)$	[3.25]
S	$0 = QS^0 - QS + \left[-\frac{\hat{q}S}{K+S}X_a\right]V$ (r_{ut})	solve for X_a →	$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x - (1+b\theta_x)}$	[3.24]
X_i	$0 = QX_i^0 - QX_i + \left[(1-f_d)bX_a\right]V$	Solve for X_i →	$X_i = X_i^0 + X_a(1-f_d)b\theta$	[3.30]
UAP	$0 = 0 - Q(UAP) + \left[r_{UAP} + r_{deg\text{-}UAP}\right]V$	Solve →	$UAP = -\frac{(\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta)}{2} + \frac{\sqrt{(\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta)^2 - 4K_{UAP}k_1 r_{ut}\theta}}{2}$	[3.38]
BAP	$0 = 0 - Q(BAP) + \left[r_{BAP} + r_{deg\text{-}BAP}\right]V$	Solve →	$BAP = \frac{-(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta)}{2} + \frac{\sqrt{(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta)^2 + 4K_{BAP}k_2X_a\theta}}{2}$	[3.39]

Rearrange mass balance.

Xa m.B. divide by $Y_a = 1$

$$0 = -1 + \left[\frac{Y\hat{q}}{K+S} - b\right]\frac{V}{Q}$$

$$(1 + b\Theta_x)K + bS = Y\hat{q}S\Theta_x \quad \text{rearrange with S on LHS}$$

$$S = K(1+b\Theta_x)$$

$$Y\hat{q}S\Theta_x - (1+b\Theta_x)$$

S m.e.

rearrange w/ X_a on LHS

$X_a =$

$$\frac{1}{\Theta_x}\left[\frac{\hat{q}}{b}\right] = \frac{(1+b\Theta_x)}{Y} \Rightarrow$$

X_a

X_i m.e.

$$X_i = \quad + (1-f_d)bX_a +$$

UAP m.B.

$$0 = -Q\,UAP - k_1 r_{ut} V - \frac{\hat{q}_{UAP}\,UAP}{K_{UAP} + UAP} X_a V$$

divide by Q, let V/Q = θ, rearrange

$$(UAP + k_1 r_{ut}\theta)(K_{UAP} + UAP) = -\hat{q}_{UAP}\,UAP\,X_a\theta$$

multiply out and move to L.H.S

$$UAP^2 + K_{UAP}UAP + k_1 r_{ut}\theta\,UAP + k_1 r_{ut}\theta K_{UAP} + \hat{q}_{UAP}UAP X_a\theta = 0$$

solve with the quadratic formula

$$a(UAP)^2 + b\,UAP + c = 0$$

$$UAP = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$, $b = K_{UAP} + k_1 r_{ut}\theta + \hat{q}_{UAP} X_a\theta$, $c = k_1 r_{ut}\theta K_{UAP}$

since answer cannot be negative, only valid solution

$$UAP = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \xrightarrow{\text{yield}} \text{eqn } [illegible]$$

$$r_{UAP} = -k_1 r_{ut}$$

$$r_{deg\text{-}UAP} = -\frac{\hat{q}_{UAP}\,UAP}{K_{UAP} + UAP} X_a$$

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$$0 = -Q\,BAP \ldots V - \frac{\hat{q}_{BAP} \ldots}{K_{BAP} + BAP} X_a V$$

divide by Q, let V/Q = θ, rearrange

$$(BAP - k_2 X_a \theta)(K_{BAP} + BAP) = -\hat{q}_{BAP} BAP\, X_a \theta$$

rearrange

$$BAP^2 + (-k_2 X_a \theta + K_{BAP} + \hat{q}_{BAP} X_a \theta) BAP - k_2 X_a \theta K_{BAP} = 0$$

$a = 1$, $b = -k_2 X_a \theta + K_{BAP} + \hat{q}_{BAP} X_a \theta$, $c = -k_2 X_a \theta K_{BAP}$

$$BAP = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \overset{\text{yields}}{\Rightarrow} \text{ eqn } 3.34$$

$$r_{BAP} = k_2 X_a$$

$$r_{deg\text{-}BAP} = -\frac{\hat{q}_{BAP} BAP}{K_{BAP} + BAP} X_a$$