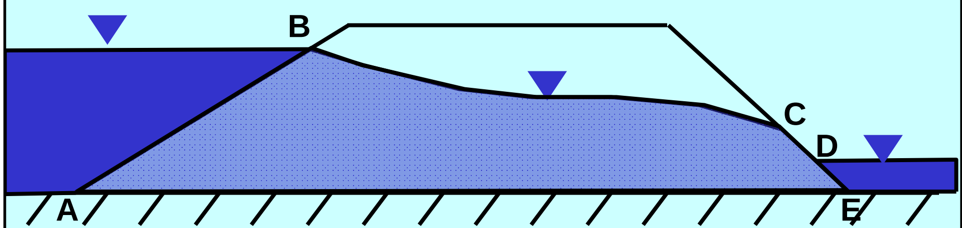
GROUNDWATER FLOW NETS

Graphical Solutions to the Flow Equations

One family of curves are flow lines
Another family of curves are equipotential lines



Boundary Conditions

AB and DE - constant head
BC - atmospheric P - phreatic surface
CD - atmospheric P - seepage face
AE - no-flow

Basic Assumptions for Drawing a Flow Net:

- material zones are homogeneous

- isotropic hydraulic conductivity

- fully saturated

- flow is steady, laminar, continuous, irrotational

- fluid is constant density

- Darcy's Law is valid

- Drawn parallel to flow

Flow into the zone between 2 flow lines = flow out of the zone

Rules for drawing flow nets

- equipotential lines parallel constant head boundaries

- flow lines parallel no-flow boundaries

- streamlines are perpendicular to equipotential lines

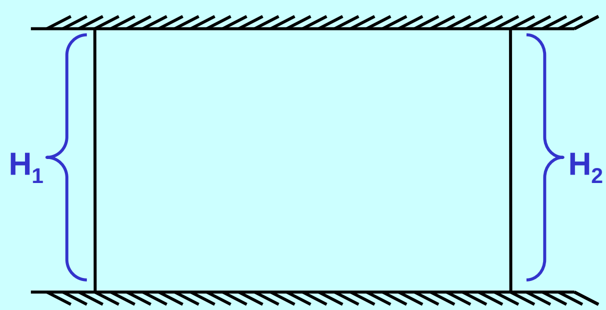
- equipotential lines are perpendicular to no-flow boundaries

- the aspect ratio of the shapes formed by intersecting stream and equipotential lines must be constant
e.g. if squares are formed, the flow net must be squares throughout
(areas near boundaries are exceptions)

Each flow tube will represent the same discharage: Q = KiA

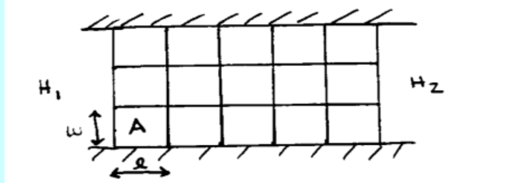
Procrastination is common. It is best to "dive in" and begin drawing. Just keep an eraser handy and do not hesitate to revise!

Draw a very simple flow net:



- equipotential lines parallel constant head boundaries
- flow lines parallel no-flow boundaries
- streamlines are perpendicular to equipotential lines
- equipotential lines are perpendicular to no-flow boundaries
- Interescting equipotential and flow lines form squares

Here is a simple net with:
4 stream lines
3 flow tubes n_f
6 equipotential lines
5 head drops n_d



Rate of flow through 1 square: $q_A = K\, i_A\, a_A$

headloss in A is $\frac{H_1 - H_2}{n_d} = \frac{H}{n_d}$ H is total head loss

$i_A = \frac{H}{l\, n_d}$ $a_A = w\,(1)$

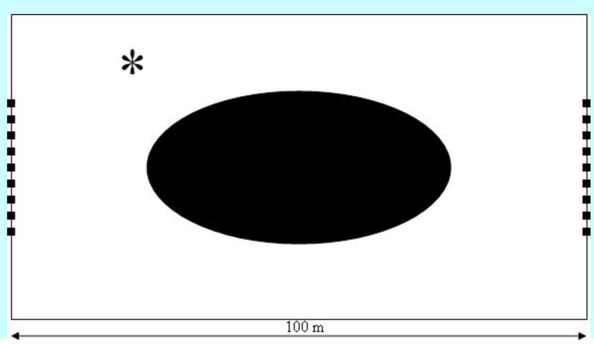
so for a unit width $q_A = \frac{KHw}{l\, n_d}$

since A is square w = l $q_A = \frac{KH}{n_d}$

Total Q per unit width = $Q = q_A n_f = KH\frac{n_f}{n_d}$

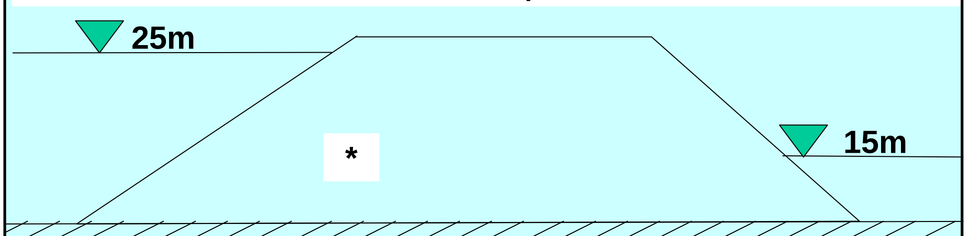
Consider an application:

A sand filter has its base at 0 meters and is 10 meters high. It is the same from top to bottom. A plan view, to-scale diagram of it is shown below. There is an impermeable pillar in the center of the filter. Reservoirs on the left and right are separated from the sand by a screen that only crosses a portion of the reservoir wall. The head in the inlet reservoir on the left is 20 m and the outlet reservoir on the right is 12m. Properties of the sand are: $K=1\times10^{-3}$ m/s $S=1\times10^{-3}$ SY=0.2. Draw and label a flow net. Calculate the discharge through the system using units of meters and seconds. What is the head at the location of the * at the top of the tank? What is the pressure at that location?



- equipotential lines parallel constant head boundaries
- flow lines parallel no-flow boundaries
- streamlines are perpendicular to equipotential lines
- equipotential lines are perpendicular to no-flow boundaries
- form squares by intersecting stream and equipotential lines

Try this before next class
K = 0.53m/day
Draw the flow net
Calculate Q
What is the maximum gradient?
What are the head and pressure at the *?



We can use the flow net to identify areas where critical gradients may occur and determine the magnitude of the gradient at those locations

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Stress caused in soil by flow = j = $i\gamma_w$
If flow is upward, stress is resisted by weight of soil
If j exceeds submerged weight of soil, soil will be uplifted

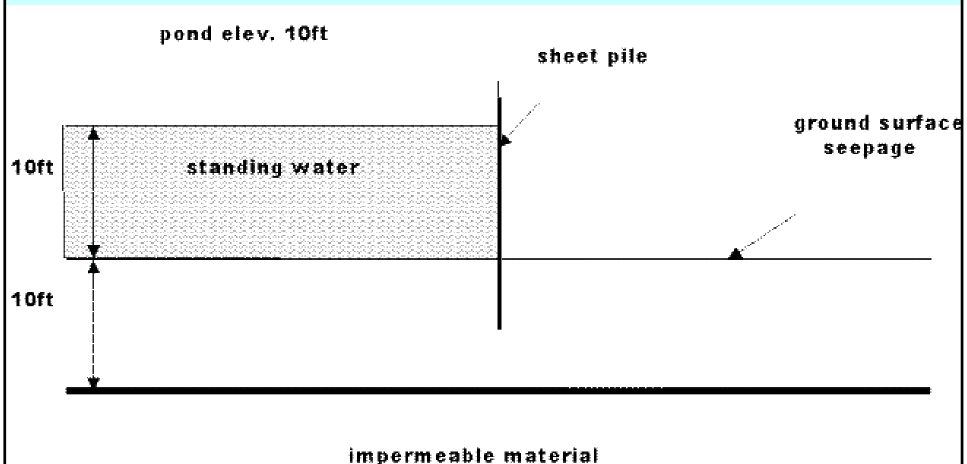
For uplift to occur j > $\gamma_{submerged\ soil} = \gamma_t - \gamma_w$
where: γ_t - unit saturated weight of soil
γ_w - unit weight of water
then for uplift to occur:
$i\,\gamma_w > (\gamma_t - \gamma_w)$
the critical gradient for uplift then is: $i_{\text{critical}} = \dfrac{\gamma_t - \gamma_w}{\gamma_w}$

What is the critical gradient for a soil with 30% porosity and a particle density of 2.65 g/cc (165 lb/ft^3)?

We can use the flow net to identify areas where critical gradients may occur and determine the magnitude of the gradient at those locations

What is the flux under the sheet pile wall if K=2ft/day? Will piping occur?

$$Q = q_A n_f = KH\frac{n_f}{n_d}$$



A PLAN VIEW FLOW NET BY CONTOURING USING FIELD HEADS AND DRAWING FLOW LINES PERPENDICULAR: can't assume constant K or b

assuming no inflow from above or below, we can evaluate relative T:

$$Q = A_A V_1 = A_B V_2$$

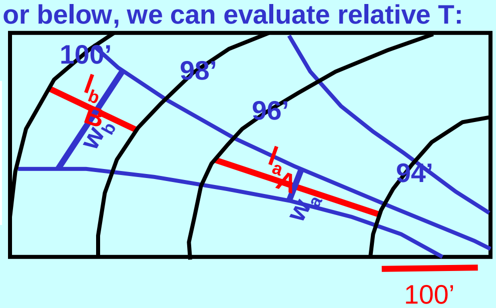
$$A_A K_A \frac{\Delta h}{l_A} = A_B K_B \frac{\Delta h}{l_B}$$

$$\frac{A_A K_A}{l_A} = \frac{A_B K_B}{l_B} \qquad \frac{K_A}{K_B} = \frac{A_B l_A}{A_A l_B}$$

$$A = wb \quad (b = \text{aquifer thickness})$$

$$\frac{K_A}{K_B} = \frac{w_B b_B l_A}{w_A b_A l_B}$$

$$\frac{K_A b_A}{K_B b_B} = \frac{w_B l_A}{w_A l_B} = \frac{T_A}{T_B}$$



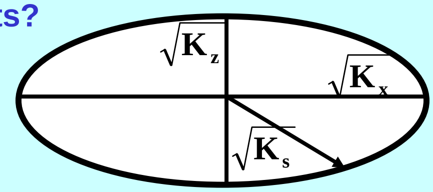
"Irregularities" in "Natural" flow nets

- varying K
- varying flow thickness
- recharge/discharge
- vertical components of flow

Nature's flow nets provide clues to geohydrologic conditions

ANISOTROPY: How do anisotropic materials influence flownets?

Conductivity Ellipsoid:



Flow lines will not meet equipotential lines @ right angles,
but they will if we transform the domain into an equivalent isotropic section,
draw the flow net, and transform it back.

For the material above, we would either expand z dimensions or compress x dimensions
To do this we establish revised coordinates

In the case where Kz is smaller: stretch z

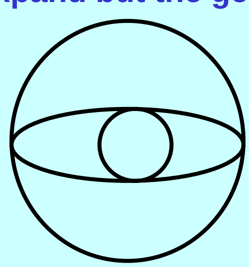
$$x' = x \quad z' = \frac{z\sqrt{K_x}}{\sqrt{K_z}}$$

or shrink x

$$x' = \frac{x\sqrt{K_z}}{\sqrt{K_x}} \quad z' = z$$

Most noticeable is the lack of orthogonality when the net is transformed back

Size of the transformed region depends on whether you choose to shrink or expand but the geometry is the same.



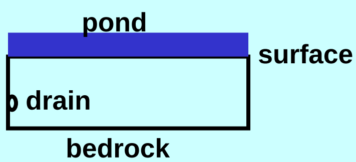
To calculate Q or V, work with the transformed sections

But use "transformed" K

$$K' = \sqrt{K_x K_z}$$

If the pond elevation is 8m, ground surface is 6m, the drain is at 2m (with 1m diameter, so bottom is at 1.5m and top is at 2.5m), bedrock is at 0m, K_x is 16m/day and K_z 1m/day, what is the flow at the drain?

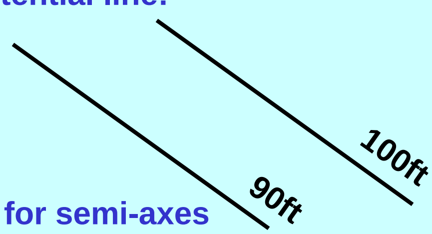
Transform the flow field for this system and draw a flow net.



$$x' = x \quad z' = \frac{z\sqrt{K_x}}{\sqrt{K_z}}$$

$$x' = \frac{x\sqrt{K_z}}{\sqrt{K_x}} \quad z' = z$$

If you want to know flow direction at a specific point within an anisotropic medium, undertake the following construction on an equipotential line:



1 - Draw an INVERSE K ellipse for semi-axes

$$\frac{1}{\sqrt{K_x}} \quad \text{and} \quad \frac{1}{\sqrt{K_z}}$$

2 - Draw the direction of the hydraulic gradient through the center of the ellipse and note where it intercepts the ellipse
3 - Draw the tangent to the ellipse at this point
4 - Flow direction is perpendicular to this line

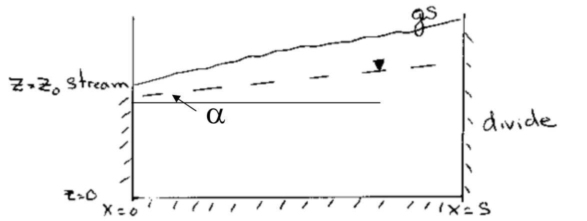
$$\text{try it above for } K_x = 16\text{ft/day} \quad \text{and} \quad K_z = 4\text{ft/day}$$

Toth developed a classic application of the Steady State flow equations for a Vertical 2D section from a stream to a divide

His solutions describe flow nets … both are methods for solving the flow equations

he solved the Laplace Equation

$$\frac{\partial^2 h}{\partial x^2} - \frac{\partial^2 h}{\partial z^2} = 0$$



boundaries

left $\quad \frac{\partial h}{\partial x}(0, z) = 0$ right $\quad \frac{\partial h}{\partial x}(s, z) = 0$

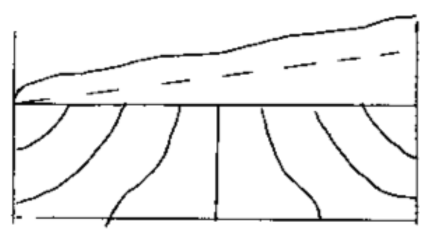
lower $\quad \frac{\partial h}{\partial z}(x, 0) = 0$

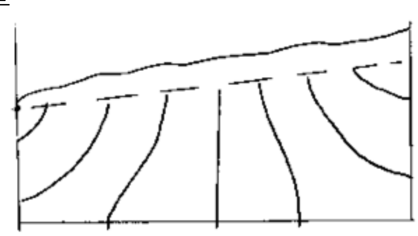
upper water table $\quad h(x, z_o) = z_o + cx = z_o + \tan(\alpha)x$

Toth's result:

$$h(x,z) = z_o + \frac{cs}{2} - \frac{4cs}{\pi^2}\sum_{m=0}^{\infty} a$$

$$a = \frac{\cos\left[(2m+1)\frac{\pi x}{s}\right]\cosh\left[(2m+1)\frac{\pi z}{s}\right]}{(2m+1)^2\cosh\left[(2m+1)\frac{\pi z_o}{s}\right]}$$





Toth's result for system of differing depth:

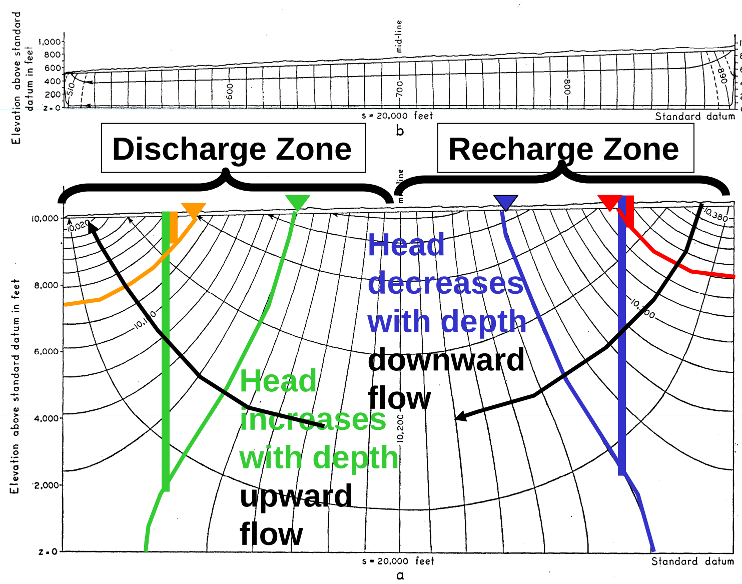


Fig. 3. Two-dimensional theoretical potential distributions and flow patterns for different depths to the horizontal impermeable boundary.

Regional Flow (Classic Papers by Freeze and Witherspoon):

homogeneous & isoptropic with and without hummocks

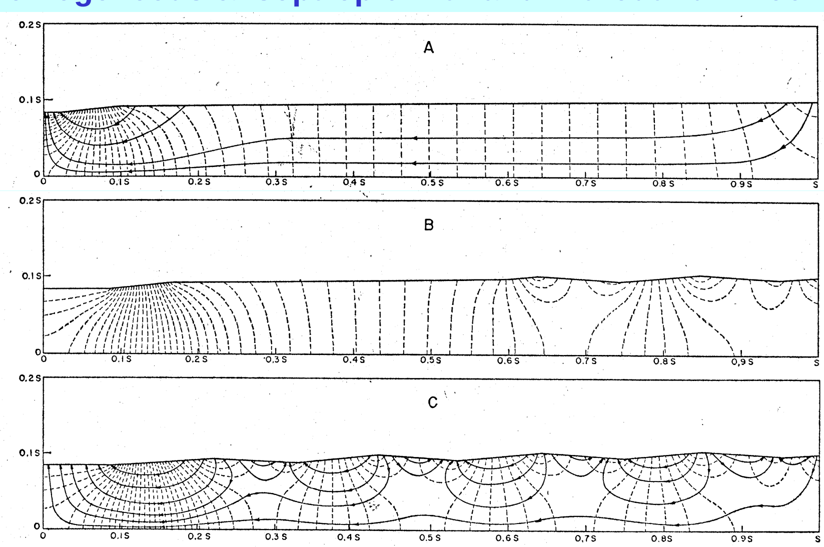
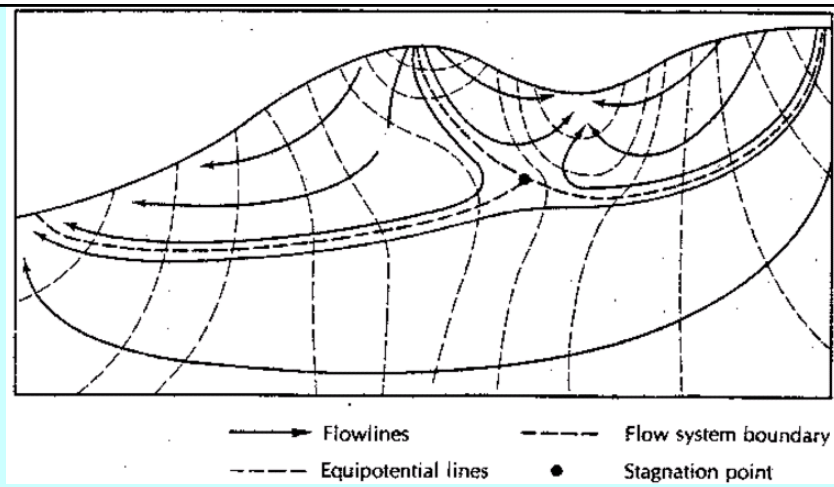


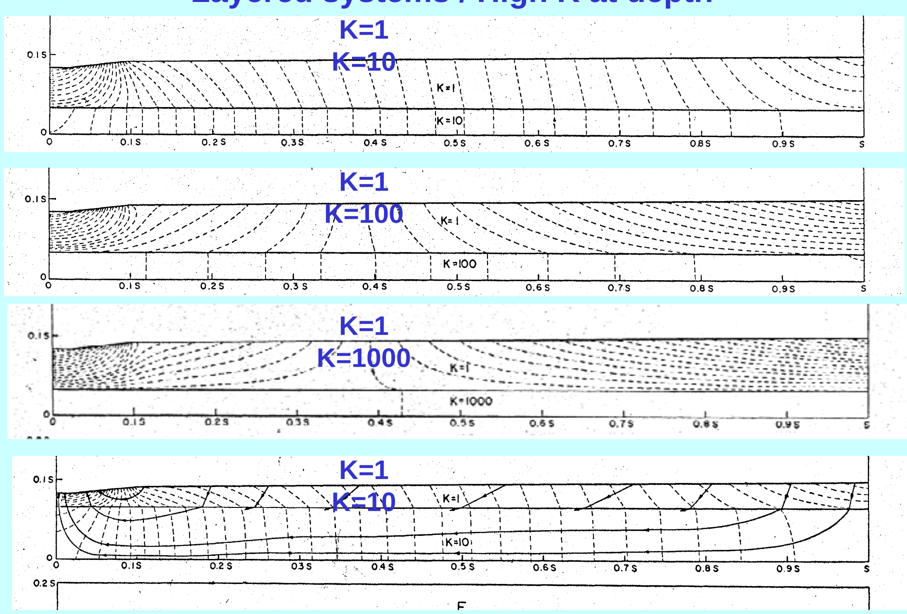
Fig. 1. Effect of water-table configuration on regional groundwater flow through homogeneous isotropic mediums.



Hierarchy of flow systems
- Local
- Intermediate
- Regional

Dominance controlled by:
- Basin Depth
- Slope of water table
- Frequency of Hummocks
- Size of Hummocks

Regional Flow (Classic Papers by Freeze and Witherspoon): Layered systems / High K at depth



Regional Flow (Classic Papers by Freeze and Witherspoon):

Layered systems / Low K at depth

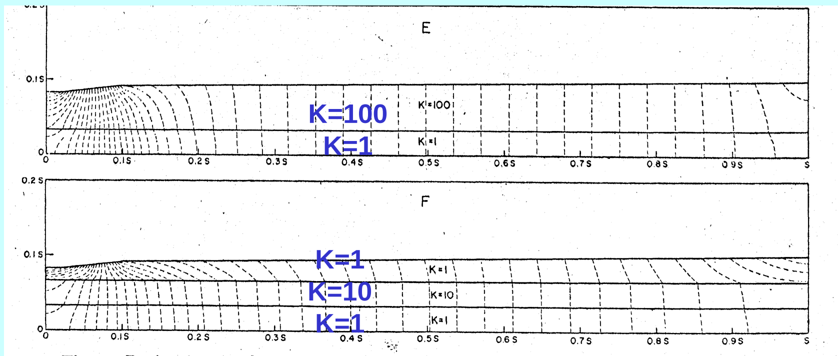


Fig. 2. Regional groundwater flow through layered mediums with a simple water-table configuration.

Regional Flow (Classic Papers by Freeze and Witherspoon):

Layered systems / High K at depth and hummocks

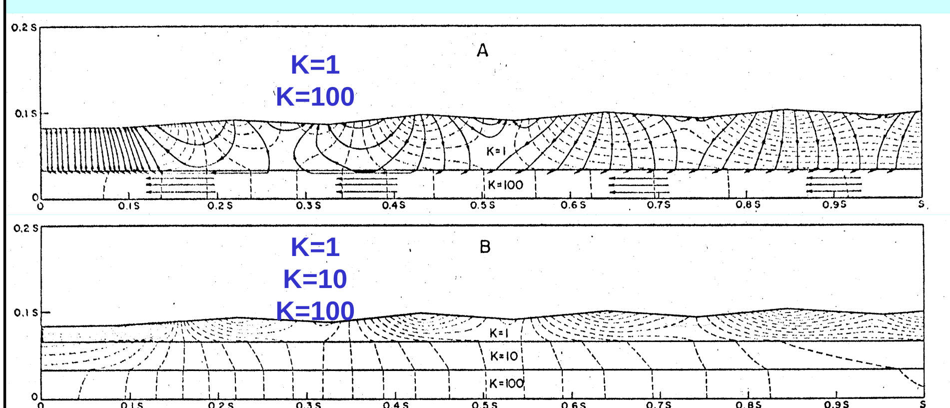


Fig. 3. Regional groundwater flow through layered mediums with a hummocky water-table configuration.

Regional Flow (Classic Papers by Freeze and Witherspoon): Partial layers and lenses

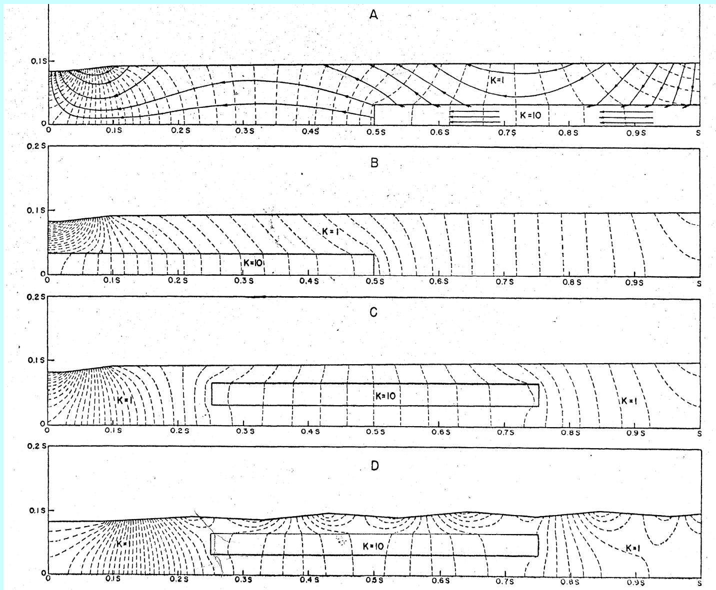


Fig. 4. Regional groundwater flow through partial layers and lenses.

K=1
K=10

K=1
K=10

K=1
K=10
K=1

K=1
K=10
K=1

Regional Flow (Classic Papers by Freeze and Witherspoon):

Layered systems / sloping stratigraphy

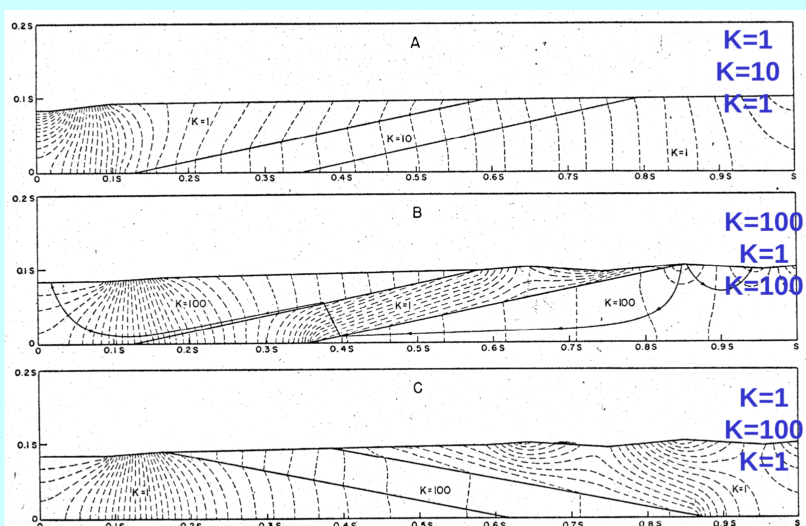


Fig. 5. Regional groundwater flow in regions of sloping stratigraphy.

Regional Flow (Classic Papers by Freeze and Witherspoon): Anisotropic systems

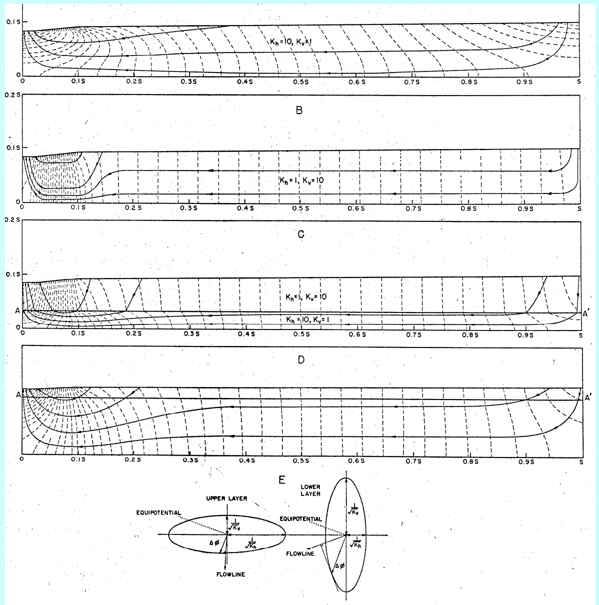


Fig. 6. Effect of anisotropy on regional groundwater flow.

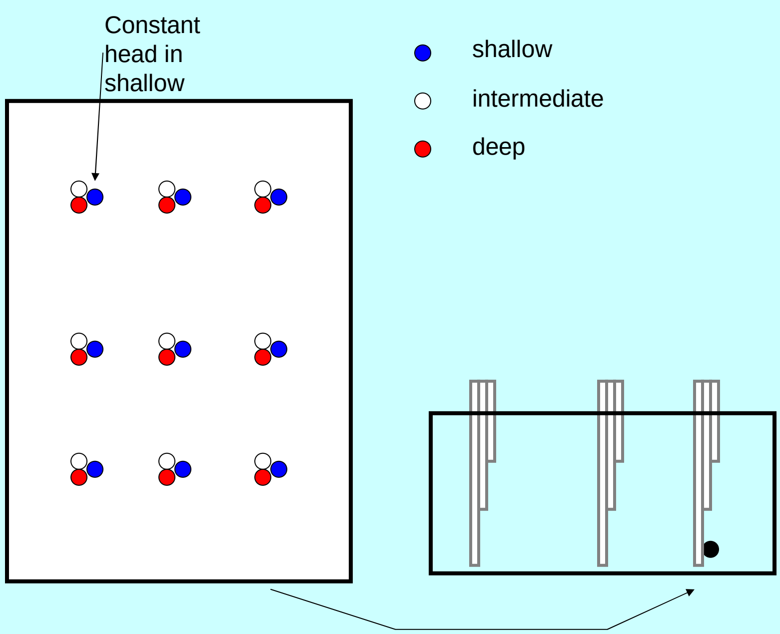
Kh=10 Kv=1

Kh=1 Kv=10

Kh=1 Kv=10

Kh=10 Kv=1

Above transformed



Explore the Flow Net Software at

http://www.mines.edu/~epoeter/_GW/11FlowNets/topodrive