Dimensional Analysis or the Factor Label Method

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Many, if not most, parameters and measurements in the physical sciences and engineering are expressed as a numerical quantity and a corresponding dimensional unit; for example: 1000 kg/m³, 100 kPa/bar, 50 miles per hour, 1000 Btu/lb. Converting from one dimensional unit to another is often somewhat complex and being able to perform such conversions is an important skill to acquire.

The method is based on quantities that can be defined as equalities.

$$\text{if } a = b, \text{ then } \frac{a}{b} = 1 \quad \text{and} \quad \frac{b}{a} = 1$$

Therefore, from any equality two fractions can be formed, each equal to one.In terms of measured quantities:

$$\text{if } 1 \text{ min.} = 60 \text{ s}, \text{ then } \frac{1 \text{ min.}}{60 \text{ s}} = 1 \quad \text{and} \quad \frac{60 \text{ s}}{1 \text{ min.}} = 1$$

As a rule of thumb your problem set up should look like this:

$$\text{Starting Units} \times \frac{\text{Desired Units}}{\text{Starting Units}} = \text{Desired Units}$$

Sometimes you will need to multiply by more than one ratio to get to your desired units, you can do this by using linking units. Your setup will look like this:

$$\text{Starting Units} \times \frac{\text{Linking Units}}{\text{Starting Units}} \times \frac{\text{Desired Units}}{\text{Linking Units}} = \text{Desired Units}$$

Important Conversion Factors

60 seconds = 1 minute
60 minutes = 1 hour
1 mile = 5280 feet
1 foot = 12 inches
2.54 centimeters = 1 inch
100 centimeters = 1 meter

Simple Example

How many wheels on 350 Ford pickups (use the equality 1 pickup = 4 tires)

-the starting units are pickups, the ending units need to be wheels.

$$350 \text{ pickups} \times \frac{4 \text{ wheels}}{1 \text{ pickup}} = 1400 \text{ wheels}$$

Advanced Example

Cancelling Units From
http://www.purplemath.com/modules/units.htm

Why did I put "1 hour" on top and "60 mins" underneath? Because I started with "80 miles per hour", so "hours" was underneath. I want "hours" to cancel off, so the conversion factor for hours and minutes needed to have "hours" on top. That meant that "60 mins" had to be underneath. And that dictated the orientation of the next factor: since "60 mins" was underneath and since I'd need "minutes" to cancel at some point, then the "1 min" (from the conversion factor for minutes and seconds) had to be on top; this in turn meant that "60 secs" had to be underneath.
And since I'm wanting a final answer of "per seconds", I *want* the seconds underneath, so this works out just right.

Continuing:

$$\frac{80 \text{ miles}}{\text{hour}} \cdot \frac{1 \text{ hour}}{60 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{2.54 \text{ cm}}{1 \text{ inch}} \cdot \frac{1 \text{ meter}}{100 \text{ cm}}$$

Cancel off the units:

$$\frac{80 \cancel{\text{ miles}}}{\cancel{\text{hour}}} \cdot \frac{1 \cancel{\text{ hour}}}{60 \cancel{\text{ mins}}} \cdot \frac{1 \cancel{\text{ min}}}{60 \text{ secs}} \cdot \frac{5280 \cancel{\text{ ft}}}{1 \cancel{\text{ mile}}} \cdot \frac{12 \cancel{\text{ in}}}{1 \cancel{\text{ ft}}} \cdot \frac{2.54 \cancel{\text{ cm}}}{1 \cancel{\text{ inch}}} \cdot \frac{1 \text{ meter}}{100 \cancel{\text{ cm}}}$$

Since the units cancel, leaving me with the "meters per second" that I need, I know the numbers must then be in the right places. So to get my answer, all I have to do is grab a calculator and simplify:

$$\frac{80}{1} \cdot \frac{1}{60} \cdot \frac{1}{60} \cdot \frac{5280}{1} \cdot \frac{12}{1} \cdot \frac{2.54}{1} \cdot \frac{1}{100} \approx 35.7632$$

This says that 80 miles per hour is equivalent to just under 36 meters per second, so:

40 meters per second is faster than 80 miles per hour.

Try these

1. How many centimeters are in 6.00 inches?

2. If it takes 2.5 minutes to complete a task, what is that same length of time in seconds?

3. Express 24.0 cm in inches.

4. If a container of water absorbs 3.4 cal of heat, what is the amount of energy absorbed (in joules)?

5. How many seconds are in 2.0 years?

ANSWERS

1	$? \text{ cm} = 6.00 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 15.2 \text{ cm}$ (to 3 significant figures)
2	$2.5 \text{ min.} \times \frac{60 \text{ s}}{1 \text{ min.}} = 150 \text{ s}$
3	$? \text{ in} = 24.0 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 9.45 \text{ in}$ (to 3 significant figures)
4	$3.4 \text{ cal} \times \frac{4.184 \text{ J}}{1 \text{ cal}} = 14 \text{ J}$
5	$? \text{ s} = 2.0 \text{ yr} \times \frac{365 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}}$ $= 6.3 \times 10^7 \text{ s}$ (to 2 significant figures)