



If we only consider advection and start with a "point" of material with $C_o=1000\text{mg/l}$

A point has no volume so can it have a concentration? So why do we say a "point"?

$$K = 0.1 \text{ cm/sec}$$

$$dh = 10 \text{ cm}$$

$$dl = 100 \text{ cm}$$

$$\phi = 0.2$$

How long will it take for the material to move 50cm?
What will the concentration be at that location at that time?

in the down gradient direction

$$\bar{v} = \frac{Kdh}{\phi dl} = \frac{0.1 \frac{\text{cm}}{\text{sec}}}{0.2} \frac{10 \text{ cm}}{100 \text{ cm}} = 0.05 \frac{\text{cm}}{\text{sec}}$$

$$d = \bar{v}t$$

$$t = \frac{d}{\bar{v}} = \frac{50 \text{ cm}}{0.05 \frac{\text{cm}}{\text{sec}}} = 1000 \text{ sec} = 0.28 \text{ hr}$$

$$C = C_o = 1000 \frac{\text{mg}}{\text{l}}$$



Suppose that source enters the up gradient end of a column
At a continuous concentration of $C_o=1000\text{mg/l}$

$$K = 0.1 \text{ cm/sec}$$

$$dh = 10 \text{ cm}$$

$$dl = 100 \text{ cm}$$

$$\phi = 0.2$$

$$\text{Dispersivity } \alpha_x = 5 \text{ cm}$$

What will the concentration be at 50 cm after 1000sec?

average linear velocity

$$\bar{v} = \frac{Kdh}{\phi dl} = \frac{0.1 \frac{\text{cm}}{\text{sec}}}{0.2} \frac{10 \text{ cm}}{100 \text{ cm}} = 0.05 \frac{\text{cm}}{\text{sec}}$$

distance traveled in 1000sec?

$$d = \bar{v}t = 0.05 \frac{\text{cm}}{\text{sec}} 1000 \text{ sec} = 50 \text{ cm}$$

By inspection we know that the concentration should be $0.5 \cdot C_o = 500 \text{ mg/l}$
But let's carry out the calculation

ALERT! ALERT! CORRECTION IN SUBSEQUENT LECTURE

Experiment with the spreadsheet

http://inside.mines.edu/~epoeter/_GW/22ContamTrans/C1d.xls

Note the values of C using only the first term and then both terms at times and locations where your intuition allows you to know the concentration.

When is use of the second term important? When does excel cause it to be in error?

Try $x = 50, 49, 51, 0, 100$ then $10, 30, 200$

Consider other times

TRY 10000sec $x = 500, 490, 510, 0, 1000$

then $x = 100, 200, 300, 400, 450$, compare 450 to 550 ?symmetrical?

Where and when can you know the correct C?

The second term is important for calculating C @ early times near the source.

$\bar{v} = 0.05 \frac{cm}{sec}$ $x = 0.05 \frac{cm}{sec} 1000sec = 50cm$ so $X = Y = Z = 0$ and we want C_{max}

$$D_x = \bar{v} \alpha_x + D^* = 0.05 \frac{cm}{sec} 5cm + 1 \times 10^{-10} \frac{m^2}{sec} \frac{10000cm^2}{1m^2} = 0.25 \frac{cm^2}{sec}$$

$$D_y = \bar{v} \alpha_x \frac{1}{5} + D^* = 0.05 \frac{cm}{sec} 5cm \frac{1}{5} + 1 \times 10^{-10} \frac{m^2}{sec} \frac{10000cm^2}{1m^2} = 0.05 \frac{cm^2}{sec}$$

$$D_z = \bar{v} \alpha_x \frac{1}{10} + D^* = 0.05 \frac{cm}{sec} 5cm \frac{1}{10} + 1 \times 10^{-10} \frac{m^2}{sec} \frac{10000cm^2}{1m^2} = 0.025 \frac{cm^2}{sec}$$

$$C = \frac{M}{8(\pi)^{\frac{3}{2}} \sqrt{D_x D_y D_z}}$$

$$C = \frac{1000mg}{8(\pi 1000sec)^{\frac{3}{2}} \sqrt{0.25 \frac{cm^2}{sec} 0.05 \frac{cm^2}{sec} 0.025 \frac{cm^2}{sec}}}$$

$$C = 0.0402 \frac{mg}{cm^3} \frac{1000cm^3}{l} = 40.2 \frac{mg}{l} \sim 40 \frac{mg}{l}$$