Uncertainty analysis

According to ISO (International Organisation for Standardisation) (sorry; they are European)

There are 2 types of uncertainty: measured and “all other,” or estimated

1. Measured uncertainty

When you have a *series of measurements* of the same quantity (taken under the same conditions, but never mind)

(a) Calculate the *mean* μ of the measurements

(i) Mean is *estimator* of *true value*

(b) Standard deviation *of the sample*

$$s = \sqrt{\frac{\Sigma (x_i - \mu)^2}{(N-1)}}$$

(i) The SD you are familiar with (*N* is no. of measurements)

(ii) Measures range of values of data

(iii) Is *not* the uncertainty (if you have a lot of data, you can locate the peak well within $\pm s$)

(c) Standard deviation *of the mean* (Baird's S_m)

$$\sigma \equiv \frac{s}{\sqrt{N}} = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N(N-1)}}$$

(i) *Is* measure of the uncertainty

(ii) Decreases slowly with increasing *N*

(d) σ is renamed *standard uncertainty*, symbol *u*

(i) Thus, $u_r = \sigma$; *r* for *random*

(ii) One *component* of uncertainty

2. Estimated (all other) uncertainties

(a) Uncertainties you cannot measure

(b) Estimate or guesstimate *greatest probable error* Δx

(i) Often 1/2 smallest scale division (least count)

0.5 mm, 0.5 mV, 0.05° (think DVM with 1-mV least count)

(c) ¡Δx is *not* an uncertainty!

(i) To calculate uncertainty, divide Δx by $\sqrt{3}$:

(ii) $u_x = \dfrac{\Delta x}{\sqrt{3}}$ (rabbit out of hat; sorry)

(iii) *Now* an uncertainty (*component* of uncertainty)

3. *When x* is function of some other variable *z*

And you can measure or estimate Δz (but not Δx)

(a) Use $\Delta x = \frac{\partial x}{\partial z} \Delta z$

(b) *Then* divide by $\sqrt{3}$

(c) Component of uncertainty is $u_x = \frac{\Delta x}{\sqrt{3}}$

(i) You cd also call it u_{xz} or $u_{x,z}$ or ... to remind yourself that it is uncertainty of *x* due to uncertainty in *z* (be inventive!)

4. *Add* components of uncertainty *in quadrature*:

$$u_c = +\sqrt{u_r^2 + u_x^2 + u_y^2 + \cdots}$$ (*combined standard uncertainty*)

(a) One or more of *u*'s may be an SDOM

(b) One or more of *u*'s may be an estimated uncertainty

(c) u_c is always positive, by convention

5. Expressing result of measurement

(a) Define *expanded* uncertainty

$$U = 2\,u_c$$

(U like u_c is *positive* by convention)

(b) Express result of measurement as

$$\mu \pm U$$

(i) $\mu =$ measured value or mean of measured values

(ii) $\mu - U \rightarrow \mu + U$ defines 95 % *confidence interval*

(iii) U typically has only 1 or 2 significant digits

6. 95 % confidence interval

(a) We think (hope? pray?) that there is 95 % chance that true value lies within 95 % confidence interval

(b) Uncertainty analysis is imprecise

(i) May tell more what you think about measurement than about measurement

(c) But we do the best we can

7. New or newish terms

Standard deviation of the mean ($s/\sqrt{N}$, where *s* is standard deviation of sample or population)

Error (deviation of single measurement from true value)

Greatest probable error (Δx)

Standard uncertainty (*u*, estimator of mean error)

Combined standard uncertainty (u_c; not *total* uncertainty)

Expanded uncertainty (U; for our purposes, $2u_c$)

95 % confidence interval (region around mean or measured value wherein we will find true value with 95 % probability (we *think*!); 5 % chance that true value lies outside 95 % C.I.)