300-01 Homework 1.0 **Terry Bridgman**

2. Your first LaTeX assignment is to use LaTeX to produce a document that replicates this as exactly as possible, with just two differences: First, replace the class section number and name above with your own. Second, make the following letter substitutions so that I know that you did not just photocopy this document: in Problems 3 and 5, change each m to n; in Problem 8, change each c to b. Your grade on this assignment will be based on how much your paper looks like this one.

3. Prove that every integer that is divisible by 6 is even.

Proof. Suppose $n \in \mathbb{Z}$ is divisible by 6. Then there is some $k \in \mathbb{Z}$ such that $m = 6k$. Therefore $n = 2(3k)$, and since $3k$ is also in $\mathbb{Z}$, this means that n is divisible by 2 and therefore that n is even. □

5. Define $A = \{n \in \mathbb{Z} \,|\, n^3 - n^2 - 6n = 0\}$. Prove that if $n \in A$ then $n = -2, 0, \mathrm{or} 3$.

Proof. Let $A = \{n \in \mathbb{Z} \,|\, n^3 - n^2 - 6n = 0\}$. Note that

$$\begin{aligned} n^3 - n^2 - 6n &= n(n^2 - n - 6) && \text{(factor out an } n\text{)} \\ &= n(n+2)(n-3). && \text{(factor the quadratic)} \end{aligned}$$

Therefore if $n \in A$ then $n(n+2)(n-3) = 0$, and therefore n must be equal to one of -2,0, or 3. □

8. Prove that if $a, b \in \mathbb{R}$ with $a \leq b$ then $[b, \infty) \subseteq [a, \infty)$.

Proof. Suppose $a \leq b$ in $\mathbb{R}$. For all $x \in \mathbb{R}$,

$$\begin{aligned} x \in [b, \infty) &\Longrightarrow x \geq b \\ &\Longrightarrow x \geq b \geq a && (b \geq a \text{ by hypothesis}) \\ &\Longrightarrow x \geq a && \text{(transitivity)} \\ &\Longrightarrow x \in [a, \infty). \end{aligned}$$

Therefore we have $[b, \infty) \subseteq [a, \infty)$. □