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Prediction of Biot's coefficient from rock-physical modeling of North Sea chalk

Casper Olsen[1], Kathrine Hedegaard[2], Ida L. Fabricius[1], and Manika Prasad[3]

ABSTRACT

We predict Biot's coefficient for North Sea chalk based on density and P-wave velocity for water-saturated chalk. We compare three effective medium models: Berryman's self-consistent model, the isoframe model, and the bounding-average method (BAM). The self-consistent model is used with two combinations of aspect ratios. In one combination, the aspect ratio is equal for pores and grains. In the other combination, the aspect ratio for grains is kept constant close to 1 and the aspect ratio for pores varies. All the models include one free parameter that determines the stiffness of the rock for a fixed porosity. This free parameter is compared with Biot's coefficient to discuss whether the free parameter is related to pore-space compressibility for North Sea chalk. We also discuss how consistent the models are between P-wave modulus and shear modulus for dry and water-saturated chalk. The acoustic velocity and the density data for dry and water-saturated chalk are all laboratory data. The isoframe model and the BAM model predict Biot's coefficient with a smaller error than the self-consistent model does. The free parameter in the isoframe model and the BAM model is related to Biot's coefficient. The free parameter in the self-consistent model is related only to Biot's coefficient for water-saturated chalk when the aspect ratios for pores and grains are equal. The isoframe and the BAM model are generally more consistent for chalk than the self-consistent model is.

INTRODUCTION

This study is based on core data from four chalk oil fields in the North Sea: Nana, Kraka, Gorm, and Valhall. North Sea reservoir chalk is a low-permeable and highly porous sedimentary rock consisting of irregular calcite grains held together by calcite cement. The cement is precipitated on grains and grain contacts after deposition of grains, and the degree of cementation varies among samples so that the resulting pore stiffness also varies.

Pore-space compressibility, a central parameter in hydrocarbon reservoir simulation, is related to Biot's coefficient, which also is used in effective-stress calculations in hydrocarbon reservoirs. Biot's coefficient can be calculated from density, P-wave velocity, and shear-wave velocity of dry chalk. However, from logging data, usually only density and P-wave velocity are known of the fluid-saturated rocks in situ. Relating acoustic wave velocity to different characteristics of a reservoir rock such as pore-fluid saturation, porosity, and lithology through effective medium models is thus useful in hydrocarbon exploration. In this study, we use three effective medium models to predict Biot's coefficient from P-wave velocity and density of water-saturated chalk.

Biot's coefficient

Biot's coefficient is related to elastic bulk modulus for dry rock with the equation

$$\beta = 1 - K_{dry}/K_0; \quad (1)$$

see, e.g., Mavko et al. (1998), where K_{dry} is the dynamic dry bulk modulus and K_0 is the dynamic bulk modulus of the matrix mineral in the rock. The upper limit for Biot's coefficient is 1, and it corresponds to a rock without cement at the grain contacts. In this case, K_{dry} has a finite small value close to zero, equal to the Reuss bound (Reuss, 1929). The lower limit for Biot's coefficient equals the porosity of the rock, and it corresponds to the stiffest possible arrangement of the grains, a Voigt bound (Voigt, 1910), in which cementation in grain contacts cannot make the rock stiffer (Fjaer et al. 1992). Biot's coefficient is related directly to pore-space compressibility

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[1]Technical University of Denmark, Department of Environment & Resources, Kongens Lyngby, Denmark. E-mail: caolse@statoil.com; ilf@er.dtu.dk.
[2]Formerly Technical University of Denmark, Department of Environment & Resources; currently at GEO, Danish Geotechnical Institute, Kongens Lyngby, Denmark. E-mail: kah@geoteknisk.dk.
[3]Formerly Department of Geophysics, Stanford University; currently at Rock Abuse Laboratory, Petroleum Engineering Department, Colorado School of Mines, Golden, Colorado, U.S.A. E-mail: mprasad@mines.edu.

through the relation

$$\beta = \frac{\varphi \cdot K_{dry}}{K_{\varphi}}; \qquad (2)$$

see, e.g., Mavko et al. (1998), where β is Biot's coefficient, φ is porosity, K_{dry} is dry bulk modulus, and K_{φ} is pore-space compressibility.

Biot's coefficient can be used to calculate effective stress in rock. A general effective-stress law was introduced by Brandt (1955), and later dealt with by Biot and Willis (1957), in which pore pressure in effective-stress calculations is multiplied by a factor called the effective-stress coefficient. Nur and Byerlee (1971) suggest an effective-stress law for bulk strain

$$\sigma' = \sigma - nP_p, \qquad (3)$$

where σ is confining stress, σ' is effective stress, and n is effective-stress coefficient. Under ideal elastic conditions, n for bulk moduli equals Biot's coefficient so that $n = 1 - K_{dry}/K_0$. This expression for n was derived analytically, and it also was suggested by Geertsma (1957).

Models

Several effective medium models have been presented in the literature. For an overview of different types of models, see, e.g., Wang and Nur (1992). To describe elasticity of an isotropic material, two elastic moduli need to be known. Therefore, a model should predict two elastic moduli based on the same input parameters in the model. Ideally, a model should depend on only one free parameter besides porosity. If more than one free parameter is included in a model besides porosity, it always will be possible to match any combination of two elastic moduli just by fitting the parameters in the model, but it will not be possible to predict the elastic moduli.

We have chosen to use three models with only one free parameter besides porosity: Berryman's self-consistent model (Berryman, 1980), the isoframe model (Fabricius, 2003; Fabricius et al., 2007), and the bounding-average method (BAM) model (Marion, 1990). The first two models are based on physical approaches, whereas the BAM model is heuristic. The model by Berryman is included because it is used widely, the isoframe model is included because it is developed based on petrographic studies of North Sea chalk (Fabricius, 2003), and the BAM model is included because it is easy to use in practical modeling. These models all can model two different elastic moduli, and they all contain only one free parameter. The free parameter in the models determines how stiff the rock is for fixed porosity, and the free parameter can be related to Biot's coefficient or to pore-space compressibility. The porosity is fixed for the different samples because the porosity for all the samples is known from laboratory measurements.

The self-consistent model by Berryman (1980) models bulk and shear modulus based on two coupled equations, and they must be solved by numerical iteration:

$$\sum_{i=1}^{N} x_i\left(K_i - K_{sc}^{*}\right)P^{*i} = 0 \qquad (4)$$

and

$$\sum_{i=1}^{N} x_i\left(\mu_i - \mu_{sc}^{*}\right)Q^{*i} = 0. \qquad (5)$$

We apply the model with ellipsoidal inclusions where P and Q are calculated with a set of equations before the equations are solved by iteration (Berryman, 1980). The *i*th material is referred to by i, and x_i is the volume fraction of the *i*th material. The geometric factors for the *i*th material are P^{*i} and Q^{*i}, and they depend only on the aspect ratio of the ellipsoids (Berryman, 1980). The bulk and shear modulus of the ith material are K_i and μ_i, and K_{sc}^{*} and μ_{sc}^{*} are effective bulk and shear modulus of the rock.

In the way we apply the model, the only free parameter is the aspect ratio used to calculate P and Q, because the porosity is fixed for the different samples modeled. In this model, the pores are isolated and no fluid flow is possible, which makes the model suitable for simulating elastic modulus obtained with ultrasonic frequency. The aspect ratio determines the elastic moduli for a given porosity. We apply the model with two combinations of aspect ratio where it is limited to only one free parameter. The first combination defines equal aspect ratio for pores and grains, and the other combination defines a constant aspect ratio for grains close to 1, whereas the aspect ratio of the pores varies. The combination in which the aspect ratio of the pores is close to 1, whereas the aspect ratio of the grains varies, is not included because it is too stiff for chalk. The aspect ratio cannot equal 1 exactly when the model is used with ellipsoids.

The isoframe model (Fabricius, 2003; Fabricius et al., 2007) is based on the Hashin-Shtrikman bounds (Hashin and Shtrikman, 1963). The basic assumption in the isoframe model is that in chalk, part of the solids is in suspension and part of the solids are held together in a frame. The part of the solids in the frame is characterized by a fraction called the isoframe (IF) value, and the part of the grains in suspension is then 1-IF. Elastic moduli are calculated based on a mixture of a solid frame and a suspension of solids. The equations below,

$$K = K_1 + \frac{f_2}{(K_2 - K_1)^{-1} + f_1(K_1 + 4/3G_1)^{-1}}$$

$$G = G_1 + \frac{f_2}{(G_2 - G_1)^{-1} + \dfrac{2f_1(K_1 + 2G_1)}{5G_1(K_1 + 4/3G_1)}}, \qquad (6)$$

define K and G where K_1 is bulk modulus of the solid part and K_2 is bulk modulus of the suspension. G_1 is the shear modulus of the solid, and G_2 is the shear modulus of the suspension. In this model, where part of the particles is in suspension, the volume fractions f_1 and f_2 are given as

$$f_1 = IF(1 - \varphi)$$
$$f_2 = (\varphi + (1 - IF)(1 - \varphi)), \qquad (7)$$

where φ is porosity. The isoframe model also can be applied as a heuristic model if critical porosity is included. Critical porosity is the transition point from grain-supported to fluid-supported rock (Nur et al., 1998). When the critical porosity, φ_c, is included, the volume fractions f_1 and f_2 become

$$f_1 = IF(\varphi_c - \varphi)$$
$$f_2 = (\varphi + (1 - IF)(\varphi_c - \varphi)). \quad (8)$$

The amount of solid in suspension and in frame can be related to how well grains are held together by cement in grain contacts. The IF parameter determines how stiff the rock is for a given porosity.

Marion (1990) suggests a heuristic model called the bounding-average method (BAM), in which elastic moduli of a given rock are constrained by the upper and lower Hashin-Shtrikman bound (Hashin and Shtrikman, 1963). In this model, stiffness of pores is expressed with the parameter ω given as the elastic modulus of the rock minus the lower Hashin-Shtrikman bound divided by the difference between the upper and lower Hashin-Shtrikman bound

$$\omega = \frac{M - M^-}{M^+ - M^-}, \quad (9)$$

where M is the measured modulus and M^- and M^+ are the lower and upper Hashin-Shtrikman bounds, respectively. The factor ω is assumed to be independent of pore-filling properties and dependent only on the stiffness of pores. The factor ω determines the stiffness of the model as the free parameter in the other models, but ω is a heuristic property, and it is therefore not correct to interpret ω as a physical property. Critical porosity also can be included in the BAM model as in the isoframe model.

Scope of study

Three effective medium models are used to predict Biot's coefficient based on P-wave velocity and density for water-saturated chalk. The free parameter in each of the three models is compared with Biot's coefficient to discuss whether the free parameter is related to Biot's coefficient. Whether the models are consistent for dry and water-saturated chalk also is discussed.

METHOD

This study is based on 39 chalk samples from the Nana field, Kraka field, Gorm field, and Valhall field in the central North Sea. For the samples from Kraka and Nana, acoustic P- and S-wave velocity and density were measured for dry and water-saturated samples, whereas measurements for the Valhall and Gorm samples were done only on dry rock as part of another study (Røgen et al., 2004). Acoustic velocities were obtained in the laboratory with pulse-transmission method on 1-in plugs placed in a rubber sleeve in a core holder from New England Research. A confining hydrostatic pressure of 7.5 MPa was applied to the samples, but the pore fluid was allowed to drain out of the samples so that pore pressure was kept at atmospheric pressure. The pulse was generated in a spike generator and transformed with a set of transducers to P- and S-waves with a center frequency at 0.7 MHz. The signal was recorded from an oscilloscope, and velocity was determined from first break. Uncertainty in measuring acoustic velocity is less than 1%, and uncertainty in determining density and porosity is also less than 1%. This makes the uncertainty so small that it is not marked with error bars in the figures in the "Results" section.

Porosity was determined with a helium porosimeter, and density was determined from the weight and volume of the samples. The change in porosity caused by the applied confining pressure during acoustic measurements is insignificant because of the low confining pressure and stiff samples. Porosities, densities, and acoustic velocities are given in Table 1, in which data from Røgen et al. (2004) are included because they have not been tabulated previously.

The texture of the samples was determined from thin sections in a microscope according to Dunham's classification (Dunham, 1962) (Table 2). In Dunham's classification, chalk is classified as packstone if it is grain supported by large grains with a matrix of fine calcite grains, wackestone if it is matrix supported with more than 10% large grains, and mudstone if chalk is matrix supported with less than 10% large grains. The carbonate content in the samples was determined by titration. The minerals in the insoluble residue were determined by X-ray diffraction (XRD), and selected elements (Si, Al, K) were determined by atomic-absorption spectrometry (AAS) to quantify content of quartz and clay. The carbonate content and the content of quartz and clay are given in Table 2.

The samples in this study were chosen to have as great a variety in mineralogical composition, porosity, and texture as possible (Table 2). Even though the North Sea chalk samples have a large variation in mineralogy, texture, and porosity, all the samples represent chalk. A large variation in sample properties was chosen to make the study more realistic for modeling of elastic properties in situ and because mineralogical composition and texture are usually not known when reservoir rocks are modeled based on logging data. The elastic moduli for the solid phase used in the models are those for calcite (Mavko et al., 1998). We use 71 GPa for bulk modulus and 32 GPa for shear modulus. Unless the noncarbonate content for chalk is high, the mineralogical composition does not have a large influence on results in modeling chalk (Fabricius et al., 2007). A large variation in porosity is needed to discuss applicability of the models for the porosity interval relevant for North Sea chalk hydrocarbon fields.

For each of the models, Biot's coefficient was predicted based on P-wave velocity and density for water-saturated chalk. This was done by calculating the free parameter in each of the models for the water-saturated P-wave modulus and using the free parameter to predict the dry bulk modulus. It is assumed that the model is consistent from water-saturated to dry chalk. Biot's coefficient then was calculated based on the predicted bulk modulus of dry chalk and bulk modulus of the matrix mineral (calcite) of the chalk (equation 1).

For each of the models, the free parameter needed to model P-wave modulus and shear modulus for dry and water-saturated chalk for each of the samples was obtained by matching each of the models to the elastic modulus calculated from the acoustic velocities and density for each chalk sample. The free parameter for P-wave modulus and shear modulus for dry and water-saturated chalk can be used to discuss how consistent the different models are.

RESULTS

Predicting Biot's coefficient based on P-wave velocity and density for water-saturated rock

For each of the models, the calculated Biot's coefficient based on density, and P- and S-wave velocity, for dry chalk is compared with the predicted Biot's coefficient based on P-wave velocity and density for water-saturated chalk (Figure 1a-d). For the isoframe model, the error between the predicted and calculated Biot's coefficient is 1–2% when Biot's coefficient is above 0.85 (Figure 1a). For Biot's coefficient in the interval of 0.7–0.85, the error is 3–7% (Figure 1a). For the BAM model, the error between the predicted and calculated Biot's coefficient is 1–2% when Biot's coefficient is above 0.85 (Figure 1b). The error is 3–8% between predicted and calculated

Biot's coefficient when Biot's coefficient is in the interval of 0.7–0.85 (Figure 1b). For the self-consistent model by Berryman (1980), the error between predicted and calculated Biot's coefficient is 5–7% when the aspect ratio for grains and pores is equal. The error is largest for the largest Biot's coefficient (Figure 1c). For the self-consistent model in which aspect ratio for the grains is kept constant, the error between predicted and calculated Biot's coefficient is in the interval of 4–15%, and the error is not systematic (Figure 1d). The isoframe model and BAM model tend to overpredict Biot's coefficient, and the self-consistent model tends to underpredict Biot's coefficient.

Table 1. Porosity, density of dry and water-saturated chalk, and acoustic ultrasonic velocities for dry and water-saturated chalk. Uncertainties in determining properties in this table are about 1%.

Sample depth (m)	Field	Porosity (%)	Density dry (g/cm^3)	Density sat. (g/cm^3)	V_P dry (km/s)	V_S dry (km/s)	V_P sat. (km/s)	V_S sat. (km/s)
2142.0	Gorm	23.1	2.08		3.83	2.38		
2160.9	Gorm	20.8	2.15		3.87	2.44		
2376.3	Kraka	32.0	1.77	2.17	2.76	1.79	2.87	1.45
2380.2	Kraka	28.8	1.93	2.23	3.01	1.91	3.05	1.57
2399.8	Kraka	19.5	2.18	2.38	4.21	2.59	4.25	2.41
2413.9	Kraka	31.7	1.85	2.18	2.99	1.89	3.05	1.55
2420.6	Kraka	23.7	2.11	2.31	4.26	2.39	4.19	2.24
2108.8	Nana	28.6	1.93	2.23	3.05	1.93	3.15	1.68
2110.0	Nana	32.6	1.83	2.17	2.76	1.75	2.89	1.47
2117.8	Nana	26.3	2.00	2.27	3.23	2.03	3.35	1.81
2120.9	Nana	29.2	1.92	2.22	3.02	1.89	3.08	1.61
2125.3	Nana	31.6	1.85	2.18	3.16	1.95	3.21	1.70
2126.8	Nana	20.4	2.16	2.37	3.02	2.02	3.45	1.77
2129.7	Nana	15.0	2.30	2.46	3.44	2.22	3.91	1.98
2134.3	Nana	29.8	1.90	2.21	3.25	1.98	3.25	1.69
2143.8	Nana	25.9	2.01	2.28	3.35	2.18	3.55	1.93
2148.4	Nana	32.4	1.83	2.17	2.99	1.86	3.21	1.64
2158.9	Nana	26.2	2.00	2.28	3.38	2.11	3.53	1.90
2169.9	Nana	32.8	1.82	2.16	3.22	1.95	3.29	1.78
2175.7	Nana	26.6	1.95	2.27	3.43	2.09	3.55	1.91
2188.1	Nana	20.7	2.15	2.37	3.72	2.28	3.94	2.11
2193.7	Nana	23.8	2.08	2.31	3.52	2.17	3.70	1.99
2112.0	Nana	34.7	1.76		2.77	1.73		
2119.4	Nana	25.5	2.01		3.37	2.09		
2146.8	Nana	24.1	2.05		4.13	2.42		
2149.8	Nana	27.1	1.97		3.61	2.21		
2160.5	Nana	28.9	1.92		3.37	2.06		
2178.9	Nana	26.4	1.99		3.11	1.98		
2177.2	Nana	25.9	2.00		3.63	2.21		
2189.4	Nana	20.9	2.13		3.98	2.33		
2195.4	Nana	25.5	2.01		3.53	2.16		
3255.7	Valhall	46	1.46		1.93	1.39		
3261.0	Valhall	48	1.41		1.86	1.22		
3265.0	Valhall	50	1.36		1.97	1.27		
2483.7	Valhall	45	1.49		2.32	1.48		
2492.0	Valhall	44	1.52		2.36	1.41		
2496.0	Valhall	40	1.62		2.77	1.75		
2498.4	Valhall	11	2.41		5.30	3.07		
2501.8	Valhall	12	2.38		5.31	3.08		

Consistency of models

Polynomial relationships between IF values from the isoframe model and Biot's coefficient as well as between the ω value from the BAM model and Biot's coefficient can be obtained for dry and water-saturated chalk (Figure 2a-d), and we find that IF value and ω value decrease with increasing Biot's coefficient (Figure 2a-d). If the figures for dry and water-saturated chalk are compared, both the isoframe model and the BAM model are consistent between dry and water-saturated chalk for P-wave modulus, but the models are not consistent between dry and water-saturated chalk for shear modulus (Figure 2a-d). For a given β, the free parameter in the isoframe and the BAM models is larger for shear modulus for dry chalk than the free parameter for shear modulus for water-saturated chalk (Figure 2a-d).

For Berryman's self-consistent model in which the aspect ratio for pores and grains are equal, the aspect ratio decreases with increasing Biot's coefficient for water-saturated chalk (Figure 3a). For dry chalk, no relationship is obtained between the aspect ratio and Biot's coefficient (Figure 3b). When the aspect ratio is constant 0.99 for grains and the aspect ratio for the pores varies, no *clear* relationship is obtained between aspect ratio for pores and Biot's coefficient for dry and water-saturated chalk, although extremely high aspect ratios are predicted for chalk with high Biot's coefficient (Figure 3c and d). Berryman's self-consistent model is consistent between two moduli for the same pore fluid but not between dry and water-saturated chalk (Figure 3a-d).

DISCUSSION

Predicting Biot's coefficient from P-wave modulus for water-saturated chalk

The different models do not predict Biot's coefficient from P-wave modulus for water-saturated chalk equally well (Figure 1a-d). How well a model predicts Biot's coefficient depends on how consistent the model is between dry and water-saturated chalk. The isoframe and BAM models might be more consistent for lower values than for higher values of the free parameter because the error between predicted and calculated Biot's coefficient is lower for higher values of Biot's

Table 2. Texture according to Dunham classification, content of carbonate, quartz, and clay of the solid phase as calculated from AAS analysis combined with XRD. The clay is dominantly smectite. *Other* represents the part of the insoluble residue not quantified as quartz and clay and includes pyrite, barite, and feldspar. W represents wackestone, M mudstone, and P packstone.

Sample depth (m)	Field	Texture	$CaCO_3$ content (%)	Quartz content (%)	Clay content (%)	Other (%)
2142.0	Gorm	W	97.9	0.8	0.9	0.5
2160.9	Gorm	W	98.6	0.6	0.5	0.4
2376.3	Kraka	M	87.2	10.9	1.2	0.7
2380.2	Kraka	M	85.3	12.5	1.4	0.7
2399.8	Kraka	W	66.6	32	0.8	0.6
2413.9	Kraka	W	83.6	14	1.6	0.9
2420.6	Kraka	W	96.3	2.6	0.8	0.3
2108.8	Nana	W	84.0	14.8	0.6	0.6
2110.0	Nana	M	87.3	11.2	0.8	0.7
2117.8	Nana	M	93.3	4.9	1.1	0.8
2120.9	Nana	M	90.4	8.0	0.9	0.8
2125.3	Nana	M	85.6	13.2	0.6	0.5
2126.8	Nana	M	84.6	12.2	1.9	1.4
2129.7	Nana	P	87.2	10	1.6	1.2
2134.3	Nana	M	93.1	4.9	1.2	0.7
2143.8	Nana	M	96.5	2	0.8	0.7
2148.4	Nana	M	97.5	1.2	0.6	0.6
2158.9	Nana	M	98.0	1.1	0.5	0.5
2169.9	Nana	M	-	-	-	-
2175.7	Nana	M	97.5	0.9	1	0.6
2188.1	Nana	W	97.1	1.5	0.7	0.6
2193.7	Nana	W	97.8	0.9	0.8	0.5
2112.0	Nana	M	94.6	4.4	0.6	0.4
2119.4	Nana	W	94.3	4.1	0.9	0.7
2146.8	Nana	M	98.6	0.9	0.2	0.2
2149.8	Nana	M	97.9	1.1	0.5	0.5
2160.5	Nana	M	98.5	0.8	0.4	0.3
2178.9	Nana	M	98	0.9	0.5	0.6
2177.2	Nana	M	98.3	0.8	0.5	0.4
2189.4	Nana	W	98	0.7	0.8	0.6
2195.4	Nana	W	98.4	0.6	0.5	0.4
3255.7	Valhall	M	98.6			
3261.0	Valhall	M	97.5			
3265.0	Valhall	M	96.6			
2483.7	Valhall	M	98.6			
2492.0	Valhall	M	98.3			
2496.0	Valhall	W	98			
2498.4	Valhall	M	98.9			
2501.8	Valhall	M	98.7			

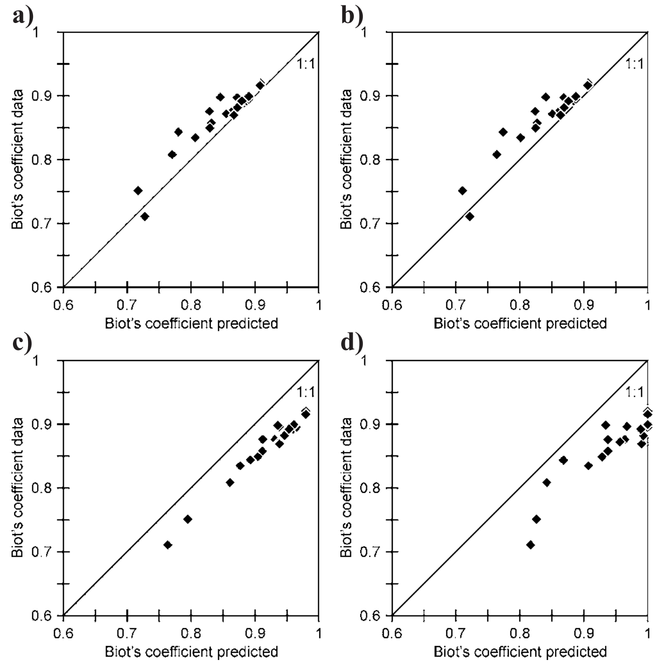

Figure 1. Biot's coefficient calculated from measured acoustic velocities and dry density versus predicted Biot's coefficient from the different models based on saturated P-wave modulus. (a) Biot's coefficient predicted based on the isoframe model. (b) Biot's coefficient predicted based on the BAM model. (c) Biot's coefficient predicted based on the self-consistent model in which the aspect ratios for pores and grains are equal. (d) Biot's coefficient predicted based on the self-consistent model in which the aspect ratio for grains is kept constant close to 1.

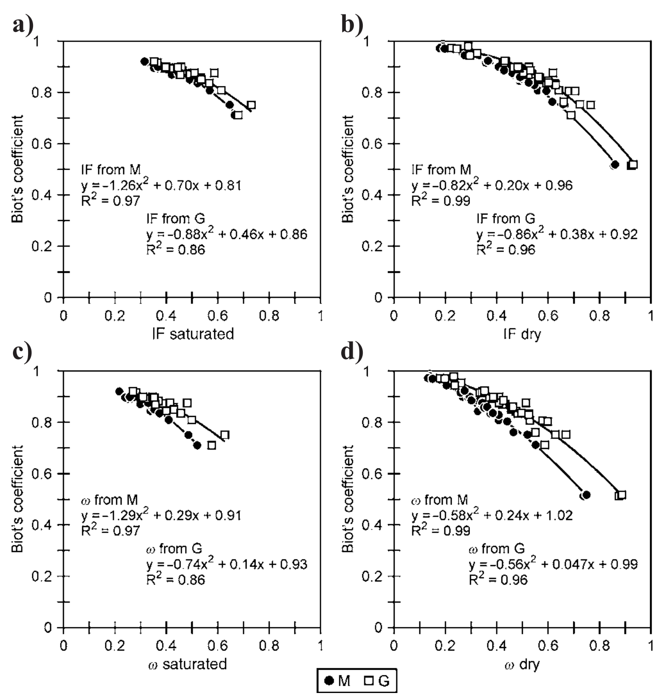

Figure 2. (a and b) Biot's coefficient versus IF from isoframe model needed to model saturated and dry P-wave modulus (M) and shear modulus (G). (c and d) The same combination of plots with ω for the BAM model. Critical porosity in all cases is equal to 1.

coefficient than for lower value of Biot's coefficient. Berryman's model, in which the aspect ratio is equal for grains and pores, underpredicts Biot's coefficient, and the error increases slightly with decreasing Biot's coefficient. In this case, the model must be almost equally consistent independent on the value of aspect ratio.

When Berryman's model is used with a constant aspect ratio for the grains and the aspect ratio of the pores varies, the model underpredicts Biot's coefficient, but the error between calculated and measured Biot's coefficient is not systematic. This model may be less consistent between dry and water-saturated chalk. The isoframe and BAM models predict Biot's coefficient better than Berryman's self-consistent model does.

Consistency of models

Consistency of the three models is discussed based on how well the free parameter relates to Biot's coefficient (Figures 2a-d and 3a-d). For water-saturated chalk, the isoframe and the BAM models are consistent between P-wave modulus and shear modulus. The same free parameter is obtained for P-wave modulus and shear modulus apart from the samples with highest value of the free parameter (Figure 2a and c). For dry chalk, the free parameter for the shear modulus is larger than the free parameter for the P-wave modulus (Figure 2b and d). From dry to water-saturated chalk, the models are consistent for P-wave modulus but not for shear modulus. The free parameter for shear modulus for dry chalk is larger than the free parameter for shear modulus for saturated chalk.

The inconsistencies can be explained by physical aspects for chalk. Røgen et al. (2005) found that shear modulus is larger for dry chalk than for water-saturated chalk. This phenomenon is described as shear weakening in the literature (Baechle et al., 2005) although, according to our models, it is rather a shear strengthening of the dry shear modulus. Strengthening of shear modulus is not included in the isoframe and BAM models, and this effect can influence the consistency of the models. The P-wave modulus is apparently less sensitive to strengthening because the models are consistent between P-wave modulus for dry and water-saturated chalk.

The self-consistent model is consistent between two moduli for the same fluid, which is to be expected because the model calculates two moduli simultaneously with iteration. The model is not consistent, however, between dry and water-saturated chalk because the aspect ratio for dry and water-saturated chalk are clearly different (Figure 3a-d). This inconsistency can be caused by a problem with the model itself, but the inconsistency also can be influenced by strengthening of dry shear modulus, as discussed for the isoframe and BAM models.

Biot's coefficient versus the free parameter in the models

For the isoframe and BAM models, a well-defined relationship is obtained between the free parameter and Biot's coefficient (Figure 2a-d). In all cases, the free parameter increases with decreasing Biot's coefficient, and for both models, the free parameter in the model is related to pore-space compressibility. For the BAM model, however, the free parameter cannot be interpreted directly as a physical parameter because the model is heuristic.

The aspect ratio in the self-consistent model is related to Biot's coefficient for water-saturated chalk, when the aspect ratios for pores and grains are equal (Figure 3a), and we find that the aspect ratio increases with decreasing porosity. If the aspect ratio increases, then the stiffness of the rock should increase, and consequently, Biot's coefficient should decrease. The aspect ratio is not related to Biot's coefficient for dry chalk when the aspect ratios for pores and grains are equal (Figure 3b). For the combination of the self-consistent model in which aspect ratio for grains is constant close to 1 and the aspect ratio of pores varies, the aspect ratio is not related to Biot's coefficient. In this case, the aspect ratio can be too abstract a parameter to relate to Biot's coefficient or pore-space compressibility. Consequently the self-consistent model predicts Biot's coefficient better when the aspect ratios for pores and grains are equal than when the aspect ratio of grains is kept constant (Figure 1c and d). Overall, a better agreement between the free parameter and Biot's coefficient or pore-space compressibility is obtained for the isoframe and BAM models than for the self-consistent model.

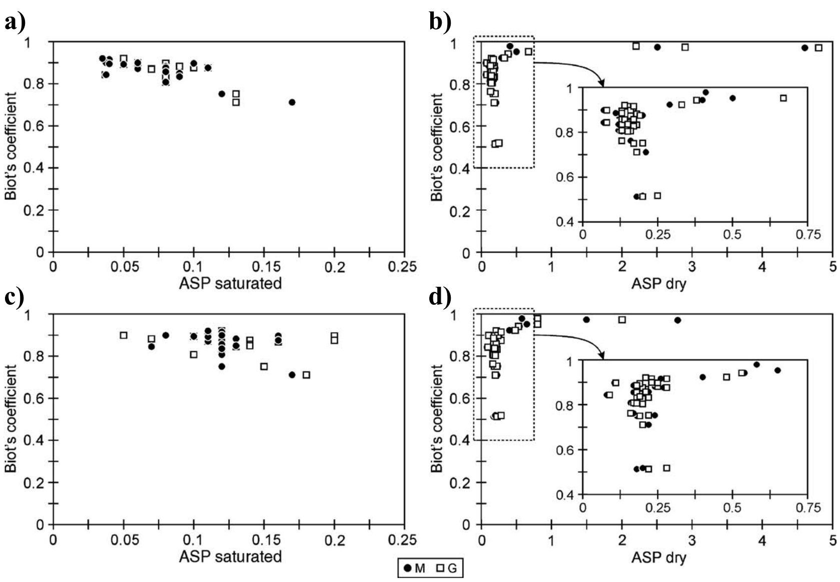

Figure 3. (a and b) Biot's coefficient versus aspect ratio needed to model saturated and dry P-wave modulus (M) and shear modulus (G) with the self-consistent model by Berryman (1980) in which the aspect ratios for pores and grains are equal. (c and d) The same combination of plots showing the aspect ratio for the pores when the aspect ratio for grains is kept constant at 0.99.

Critical porosity and heuristic models

The critical porosity (Nur et al., 1998) can be included in the Hashin- Shtrikman-based models, the isoframe model (Fabricius, 2003), and the BAM model (Marion, 1990). The BAM model is a heuristic model, and the isoframe model is turned into a heuristic model when critical porosity is included. Critical porosity is determined for the chalk in this study from where the trend line for both V_P and V_S for dry chalk intercepts the porosity axis. We obtain a critical porosity of about 75% (Figure 4). Anderson (1999) and Fabricius (2003) used the method by Nur et al. (1998) to determine critical porosity. Anderson (1999) obtained a critical porosity of 60%, and Fabricius (2003) obtained a critical porosity of 60–75%. Critical porosity of 75% for the chalk in this study is therefore in agreement with previous studies on chalk. Critical porosity can depend on texture of the chalk, but for simplicity, we apply a constant value for critical porosity.

The isoframe and the BAM models are not more consistent when critical porosity is included in the models (Figures 2a-d and 5a-d). The models therefore might not predict the Biot's coefficient more accurately based on density and P-wave velocity for water-saturated chalk when critical porosity is included. If critical porosity equal to 100% is applied in the models, they are easier to use in practical modeling.

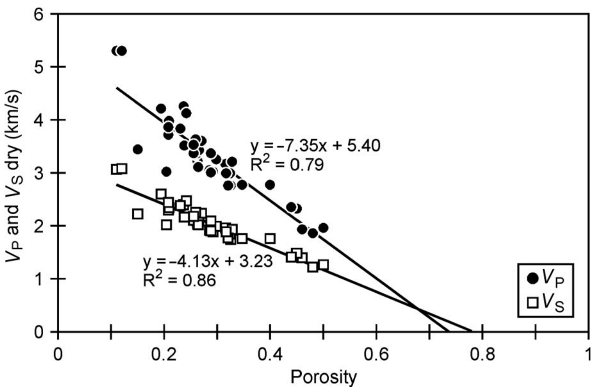

Figure 4. P-wave velocity, V_P, and shear wave velocity, V_S, for dry rock versus porosity. Critical porosity is obtained as the interception of the trend line with the porosity axis. Critical porosity ϕ_c is about 0.75.

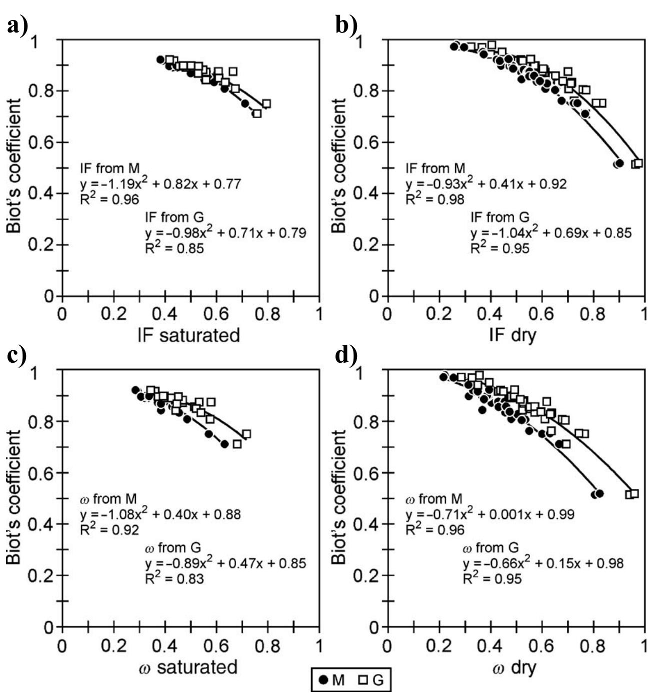

Figure 5. (a and b) Biot's coefficient versus IF from isoframe model needed to model saturated and dry P-wave modulus (M) and shear modulus (G). (c and d) The same combination of plots with ω for the BAM model. Critical porosity in all cases is equal to 0.75.

CONCLUSIONS

For the isoframe and the BAM models, the error between the predicted Biot's coefficient based on density and P-wave velocity for water-saturated chalk and the calculated Biot's coefficient is 1–2% for Biot's coefficient above 0.85, and for Biot's coefficient in the interval of 0.75 to 0.85, the error can be as much as 8%. For the self-consistent model in which the aspect ratios for pores and grains are equal, the error between the predicted and calculated Biot's coefficient is constant 5–7%. For the self-consistent model in which the aspect ratio for grains is kept constant close to 1 and the aspect ratio of pores varies, the error between predicted and calculated Biot's coefficient is 4–15% and the error is not systematic.

The isoframe model and the BAM model are consistent for water-saturated chalk between P-wave modulus and shear modulus and between P-wave modulus for dry and water-saturated chalk. The models are not consistent between P-wave modulus and shear modulus for dry chalk, and the models are not consistent between shear modulus for dry and water-saturated chalk. Including a critical porosity of 75% in the isoframe model and the BAM model does not make the models more consistent than applying critical porosity equal to 100%. The self-consistent model is consistent between P-wave modulus and shear modulus for the same pore fluid, but the model is not consistent between dry and water-saturated chalk.

The free parameter in the isoframe and the BAM models are related to Biot's coefficient or pore-space compressibility. For the self-consistent model, the aspect ratio is related only to Biot's coefficient when the aspect ratios for pores and grains are equal for water-saturated chalk.

Overall, the isoframe model and the BAM model are better to use for chalk than is the self-consistent model by Berryman. They predict Biot's coefficient better, the free parameter in the isoframe and BAM models are clearly related to Biot's coefficient, and the isoframe and BAM models are more consistent for modeling two elastic moduli for chalk.

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