

Empirical Velocity-Pressure and Porosity-Pressure Trends in Unconsolidated Sands

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Summary

We developed empirical velocity-pressure and porosity-pressure trends from compressional and shear wave velocity and porosity measurements on unconsolidated sand and glass bead samples over a pressure range from 100 kPa to 20 MPa. We found the S-wave velocity to vary with approximately the fourth root of the effective pressure over the entire range of pressures, while the P-wave velocity demonstrates a slightly lower pressure dependence. Preconsolidation reduces the pressure dependence of both velocities, with the effect being larger for the P-wave velocities, and results in largely unrecoverable porosity loss. Samples prepared with varying sorting qualities produced a range of initial porosities, but the effect on the P and S-wave velocities and on their pressure trends was minimal. Water-saturated P and S-wave velocities were modeled with Gassmann fluid substitution. The water-saturated P-wave velocities show a large porosity dependence, which results in a significant porosity dependence in the V_P - V_S ratio, especially at low pressures.

Introduction

The hazards posed to offshore drilling by unknown overpressures at shallow depths have prompted the use of P-wave velocities or V_P - V_S ratios to detect overpressures prior to or during drilling. In general, pressures are assessed by developing an empirical, site-specific, normally pressured P-wave interval-travel-time vs. depth trend. This trend is usually assumed to be linear when depth is plotted against the log of the interval travel-time, and deviations above this trend are assumed to be the result of high pore pressures (Pennebaker, 1970; Pilkington, 1988). Recently Huffman and Castagna (2001) and Prasad (2002) have demonstrated the potential of the V_P - V_S ratio as an indicator of pressure, which may be extracted from multi-component or large-offset reflection data. The V_P - V_S ratio is especially sensitive to pressure for water-saturated sediments. At low effective pressures the shear velocity drops to near zero while the P-wave velocity is limited to values above 1500 m/s, and so the V_P - V_S ratio rises dramatically at low pressures.

Because of the frequent use of the dynamic shear modulus in geotechnical applications, a great deal of research has been conducted on the pressure dependence of the shear modulus and of the shear wave velocity in soils at very low pressures. A large body of experimental work at pressures generally below 500 kPa has demonstrated that the shear velocity of sands demonstrates a pressure dependence of

approximately the fourth root of the pressure: $p^{1/4}$ (Hardin, 1980; Hryciw and Thomann, 1993).

Hardin and Blandford (1989) developed semi-empirical expressions for the shear modulus and P-wave modulus as functions of the effective pressure and porosity. A simplified form for an isotropic stress state is given by:

$$M_{ij} = \frac{OCR^k}{F(e)} S_{ij} p_a^{1-n} \sigma'_{av} \quad (1)$$

where M_{ij} is the modulus in the plane of propagation, σ'_{av} is the mean effective stress, and p_a is the atmospheric pressure. Equation (1) includes two free parameters: S_{ij} , a multiplier to account for textural factors and structural anisotropy, and n , which dictates the pressure dependence of the modulus. The void ratio function, $F(e) = 0.3 + 0.7e^2$, is meant to account for the effect of porosity differences, whether from textural differences between samples or from the compaction of a given sample. The OCR^k term corrects the pressure dependence for the effects of preconsolidation, where OCR is the overconsolidation ratio, and k is a function of the plasticity index. The overconsolidation ratio is defined as the preconsolidation pressure divided by the current pressure, and so the pressure dependence of the modulus for unloading or reloading paths is simply the effective pressure, p , to the $n-k$. Little experimental data has been collected on unconsolidated sediments to constrain the free parameters for the P-wave modulus.

We have run a series of experiments on reconstituted sand and glass bead samples to measure the pressure dependences of the velocities in unconsolidated sands and to investigate the effects of sorting and preconsolidation on the velocities and porosity. We measured the P and S-wave velocities at pressures from below 100 kPa up to 20 MPa in order to test the pressure dependence over this entire pressure range. In this paper we report our results and discuss the porosity and velocity trends observed.

Experimental Method

The experimental apparatus consists of a sample holder that is inserted into a hydrostatic pressure vessel. The two end caps of the sample holder contain both P and S-wave transducers made with 200 kHz piezoelectric (PZT) crystals. The samples, 3.8 cm (1.5 in.) in diameter, were generally prepared to be about 3 cm long. Velocities were calculated by picking first arrivals from pulse-transmission signals. With this arrangement we were able to get interpretable shear signals at pressures as low as 50 kPa, with errors of approximately 2% for the P-wave velocities and 4% for the S-wave velocities.

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The data presented here are from samples of a fine grained, well sorted, quartz sand, called the Santa Cruz Aggregate, as well as from synthetic samples made from sieved fractions of this sand and of glass beads. Four samples of the sand were run, two dry and two water-saturated. Two other samples were made of sieved fractions of this sand, both of which were run dry: one entirely of a large grain size fraction, and a second sample of 65% by mass of the large size fraction, and the other 35% of grains about 1/4 the size (sample Sa-35%-Small). A total of seven glass bead samples were run, all dry. Three samples consisted of different narrow size ranges of beads. Three samples were made with a “bimodal” mixture of grain sizes, with 35% of the mass made up of smaller grains and 65% of larger grains. Finally, one sample was made up of a broad range of particle sizes. More detailed descriptions of the samples and of the sample preparation procedures can be found in Zimmer et al. (2002a,b).

The initial porosity of the samples was calculated from the grain density, dry sample mass, and sample volume. The changes in the sample volume and porosity with pressure were then monitored by measuring changes in the length and circumference of the samples. An error analysis of our porosity measurements estimates the error at 0.012 to 0.016, or 3 to 4% of the total porosity.

The pressure paths followed generally included a number of pressure cycles of increasing peak pressure for each cycle. The velocities and porosity were measured at the same set of pressures during each cycle (e.g. 0.1, 0.2, 0.5... MPa). This allowed us to compare the velocities and porosities measured at the same pressure for a sample that had been preconsolidated to a range of higher pressures. Five of the samples were cycled through 8 or 9 cycles, while the rest were cycled through between 1 and 5 cycles.

Pressure Trends and Preconsolidation Effects

We treated n , k , and S as free parameters and fit Equation (1) to the moduli calculated from our velocity measurements. We show the empirical fit for the two moduli from sample Sa-35%-Small in Figure 1. These figures demonstrate that there is a slight preconsolidation effect to the samples, where the trends for the unloading and reloading paths can be seen to diverge slightly. The coefficients for this sample and for the dataset as a whole are given in Table 1. The coefficients of all samples are given in Zimmer et al. (2002b). For the shear modulus we found the value of n to vary from between 0.352 to 0.612, so the pressure dependence of the S-wave velocity then varies between p to the 0.176 and the 0.306. The value of k for the shear modulus varied between -0.073 and 0.101 and averaged -0.003 (effectively zero). These values of k are relatively small compared to the values predicted for clays,

which can be as large as the value of n for high plasticity clays (Hardin and Drnevich, 1972). The pressure dependence of the P-wave modulus is consistently lower than that of the shear modulus, with n ranging from 0.304 to 0.478 and averaging 0.06 less than the n for the shear modulus for the same sample. The value of k , on the contrary, was almost always higher for the P-wave modulus than for the shear modulus, averaging 0.026 and ranging from -0.033 to 0.104 . We did not observe a significant systematic initial porosity effect on the pressure dependence (n) or on the preconsolidation dependence (k), thus the variation in initial porosity that comes from the differences in sorting does not significantly affect these dependences over the porosity range tested.

Porosity reduction of the samples was fit with an empirical expression of the form:

$$\phi = \phi_0 (1 - A p^m OCR^l) \quad (2)$$

where ϕ is the porosity, ϕ_0 is the initial porosity of the samples, p is the effective pressure, OCR is the overconsolidation ratio, and A , m , and l are free parameters. The fit of this expression is shown in Figure 2, where $1 - \phi/\phi_0$ is plotted against the pressure, and the coefficients for sample Sa-35%-Small are given in Table 1. Here the unloading path can be seen to be very different from the loading path, and most of the porosity is not recovered during unloading.

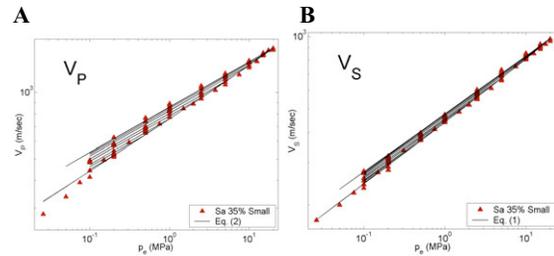


Figure 1: The velocity results from sample Sa 35% Small: A) V_P vs. pressure, B) V_S vs. pressure, The black lines represent fits to the Equation (1).

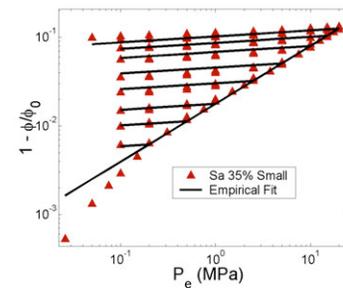


Figure 2: Normalized porosity-pressure fit for sample Sa-35%-Small, after Equation (2).

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Table 1

Sample	S	n	k	A	m	l
Sa 35% Small P-wave Mod.	1754	0.47	0.059	0.0039	0.65	0.59
Shear Mod.	539	0.51	0.029			
Entire Dataset P-wave Mod.	2336	0.41	0.032			
Shear Mod.	604	0.47	0.001			

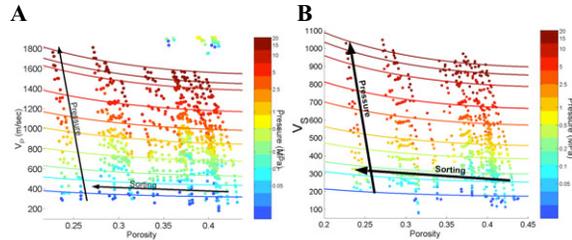


Figure 3: The velocity data from all of the samples plotted against porosity and color-coded according to the pressure at the time of measurement: A) V_p B) V_s . The lines represent the Reuss average between the moduli of the high-porosity dry-frame end-member and solid quartz.

Sorting Effect

Figure 3 shows the velocity data from all of the samples plotted together against porosity and color coded according to the pressure. These plots illustrate the relative effects of the pressure and sorting on the velocity and porosity. Increased pressures result in large increases in the velocity for these coarse sediments, but the porosity is only slightly reduced. On the contrary, the porosity reductions that result from a decrease in the sorting quality produce only minor increases in the velocity.

The lines shown in Figure 3 demonstrate the velocity effect predicted for the sorting-induced porosity changes by the Reuss average between the dry frame moduli and the solid quartz moduli. The Reuss (isostress) average, the weighted harmonic average between two end member moduli, simulates the weakest possible way to combine two materials. Here the Reuss average is used to represent the theoretically minimal possible change in the velocities produced by adding solid grain material (quartz) to the high porosity sample (assuming constant packing) (Dvorkin and Nur, 1996). The high porosity end-member moduli come from the highest porosity data for each pressure. The fact that the trend is so well described by the Reuss average implies that the small grains added tend to sit more or less passively in the pores, reducing the porosity but not adding significantly to the stiffness of the sediment.

Figure 4 shows the results of Gassmann fluid substitution with water of the P-wave velocities of all the dry samples. As seen in this figure, the water saturated P-wave velocities

have a significant porosity dependence, with lower porosities correlating to higher P-wave velocities. The presence of water in the pores stiffens a low porosity sediment more than a high porosity one, though the trend is still described reasonably well by the Reuss average between the moduli from the water-saturated, high porosity end-member and solid quartz. The shear wave velocities, which only require a density substitution, show a slight porosity dependence at the higher pressures, but no porosity dependence at low pressures.

Implications for Pressure Prediction and Monitoring

The V_p - V_s ratios for all of the Gassmann fluid-substituted data are shown in Figure 5, plotted against pressure and color-coded by porosity. The V_p - V_s ratio increases from below 3 at 10 MPa to a mean value of about 7 at 0.5 MPa. Unfortunately there is a considerable amount of scatter in the V_p - V_s ratio, especially at low pressures, that would generate a significant amount of uncertainty in the in situ effective pressure determined from V_p - V_s ratio measurements. For example, if the measured V_p - V_s ratio is 5, the in situ pressure (based on our fluid-substituted data) could vary from between 0.2 and 2 MPa.

Figure 5 demonstrates that there is a porosity dependence to much of this scatter in the V_p - V_s ratio at low pressures. Since these non-cohesive sediments demonstrate only relatively small changes in both the velocity and porosity with compaction, preconsolidation has only minor influences on the V_p - V_s ratio. This implies that for water-saturated sands there should be only very little difference between the V_p - V_s ratio signatures of overpressures produced through undercompaction or through repressurization mechanisms such as fluid expansion. As preconsolidation can cause a much larger reduction in the porosity of clay-rich sediments (Bowers, 1995), this effect could potentially be more important for cohesive sediments.

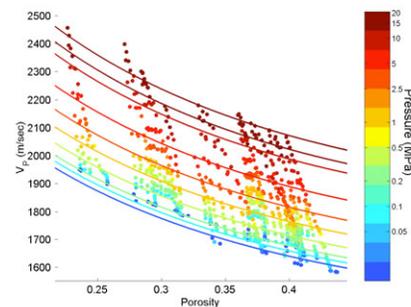


Figure 4: The P-wave velocity results after Gassmann fluid substitution with water, plotted against porosity, again showing the Reuss averages between the high porosity end-member and solid quartz.

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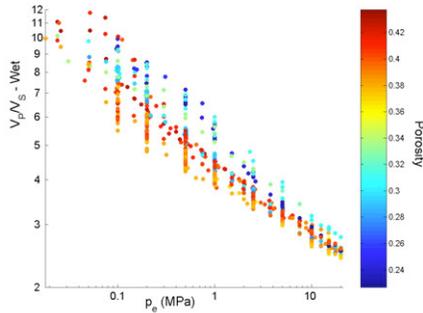


Figure 5: The V_p - V_s ratio plotted in a log-log plot against pressure and color coded according to the porosity.

The porosity variation between the samples comes primarily from the differences in the initial porosities of the samples due to differences in their sorting. The scatter in the V_p - V_s ratio is therefore a product of the different effects that the porosity has on the water-saturated P- and S-wave velocities. The small porosity-dependence of the S-wave velocities at lower pressures allows the porosity dependence of the water-saturated P-wave velocity to show through. The implication is that for robust pressure estimates to be made from measured V_p - V_s ratios it will be necessary to correct for this porosity effect at pressures below about 10 MPa. Such a correction could potentially be developed through a suite of laboratory measurements on natural sands, and would need to be developed in order to permit the use of the V_p - V_s ratio as a quantitative pressure indicator.

Conclusions

We have measured the pressure dependence of the P and S-wave velocities through sand and glass bead samples with a variety of initial porosities produced by varying their sorting. We also ran the samples through a series of pressure cycles of increasing peak pressures in order to observe the effects of preconsolidation on the samples. We have observed the following results:

- 1) The S-wave velocities vary with the fourth root of the effective pressure over the entire pressure range measured (100 kPa to 20 MPa). For the P-wave velocities the pressure dependence is slightly lower – about $p^{0.22}$.
- 2) These non-cohesive sediments do exhibit a small preconsolidation effect on the velocities, though the effect is generally smaller for the S-wave velocities than for the P-wave velocities.
- 3) While poorer sorting can result in large decreases in the porosity, it has a relatively small effect on the S-wave velocities and dry P-wave velocities. The porosity reduction does produce a larger effect on the water-saturated P-wave velocities, as calculated from Gassmann fluid-substitution of our dry velocity results. This porosity

dependence of the water-saturated P-wave velocities, and the limited porosity dependence of the S-wave velocities, results in a porosity dependence in the V_p - V_s ratio. Higher porosities correspond to a lower V_p - V_s ratio.

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