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ABSTRACT

Jointly produced renewable resources, such as fish, have been extensively studied in the literature on multi-product firms. However, relatively little empirical work has been done on jointly produced non-renewable resources such as metals. This difference is key because theoretical Hotelling-style models have shown that non-renewable resource producers faced with fixed capacity constraints and heterogeneous resources may reduce output in response to higher price, in contrast to the behavior of nearly all other types of firms. The joint product relationship is tested econometrically for five metals using a panel representing more than 100 mines across the time period 1991-2005. The estimation strategy is drawn from joint production theory, namely a flexible form, dual revenue approach with seemingly unrelated regression (SUR) estimation. The results confirm that multi-product mines respond (in the short run) to higher prices of a particular metal by reducing output of that metal. The results also show certain metals are complements and certain metals are substitutes in supply, which is arguably a more complete framework than conventional wisdom on joint metal production. The results have important implications for future modeling efforts related to metal markets.

Keywords: multi-product, joint production, mining, metal supply

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1 Introduction

Technologies that produce more than one output are common across a variety of applications in economics. When the inputs for these technologies cannot be allocated to particular outputs, the process is considered joint production. There is a large body of literature on the implications of joint production for renewable resources such as fish and agricultural goods, but only limited empirical treatment has been given to jointly produced non-renewable resources, such as minerals. In the absence of such empirical work, conventional wisdom surrounding the behavior of multi-metal firms has been left largely untested.

This paper presents three potentially surprising findings about the behavior of multi-product mining firms. The results are drawn from estimating average short-run own and cross-price elasticities of supply for five commonly joint-produced commodities (silver, gold, copper, lead, and zinc). The empirical model, a flexible-form dual revenue function, follows the approach of joint-production literature where it has been used extensively in the study of multi-product fisheries.[1] This study is the first identified attempt to estimate flexible functional forms for multi-product metal producers. The results of this model are verified and expanded using alternative reduced-form specifications.

The first finding is that mineral supply, unlike the supply of most products, decreases when prices increase. Going as far back as the late 1930's, Keynes (1936) and Paish (1938) questioned whether the common case of positively sloped short-run supply curves is valid for mining. Several theoretic and empirical studies have built on this observation to show that increasing metal prices can lead to reductions in short run supply. Slade (1988); Krautkraemer (1989); Farrow and Krautkraemer (1989); Shinkuma and Nishiyama (2000); Cairns and Shinkuma (2003) all show this result theoretically, and attribute it, generally speaking, to the heterogeneous nature of resource quality and uncertainty in future prices. In mining industry terminology, this effect would be explained through the notion of a changing "cut-off grade." A mine's cut-off grade is threshold for the lowest quality resource that the firm can profitably extract given current costs and prices. The cut-off grade changes due to market conditions and the physical characteristics of the resource. When prices increase, by definition, the mine's cut-off grade is lowered, marginal resources are extracted, and due to fixed capacity, the output of the mine falls. Marsh (1983a); Slade (1988); Farrow and Krautkraemer (1989); Shinkuma and Nishiyama (2000) show the negative elasticity of supply effect empirically. This study, using the largest and most diverse sample of mining firms to date, provides additional evidence that the short run supply curve for metals is negatively sloped.

The second finding is that mineral pairs can be either complements or substitutes in supply. In other

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[1] See Jensen (2002) for a survey of this literature.

words, for a given multi-product mine, a price increase in one mineral can lead to either a supply increase or supply decrease in the other minerals. This finding is potentially surprising given that past research, Marsh (1983b); Farrow and Krautkraemer (1989), finds the minerals in their studies are only complements in supply. The complements-only relationship is also an assumption found in Pindyck (1982), for example. Underling the complements-only relationship is the assumption that resources at a particular mine are homogeneous, and do not vary in quality across the area of the resource. This study finds that metal pairs can behave as substitutes or complements in supply, much in the same way as fish or agricultural goods do because of output targeting.

The third finding of this paper is that the own and cross-price elasticity of supply for the five metals studied seem independent of the revenue share of the respective metals. This finding is in potential conflict with a framework that is quite pervasive in discussion of mineral joint production, and is related to the notion of "main products", "co-products", and "by-products." For a particular mine, a main product is the metal that entirely determines the operation's economics, a by-product is a metal which is inconsequential to the operation's economics, and co-products are several metals that all determine the operation's viability (Tilton, 1990). Typically this framework is extended to imply that main products will be price responsive, while by-products will be relatively price inelastic. The results of this paper show two metals, which would likely be classified as by-products on the basis of their revenue, are price elastic, while two metals which might be classified as co-products are price inelastic. These findings dispute the method of using revenue share to classify main products, co-products, or by-products.[2]

The need for a more complete understanding of jointly-produced metals comes as a sharp increase in the demand for specialized materials has created concern about the ability of producers in key industries to secure inputs. Supply concerns around so called "critical materials" for end uses such as clean energy technology and national defense have lead to many studies on critical materials and their supply chains.[3] A general finding of these studies is that many critical materials are produced as joint products, and that the joint product nature is an important determinant of these materials' supply. Understanding the degree of metal supply interdependence can guide future researchers in creating more complete models of metal supply.

Because of the counter-intuitive nature of the negative own price effect (the first finding), an overview

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[2] A revenue share method is used by Nassar et al. (2015) to classify the share of global production produced as a by-product.

[3] See Bauer et al. (2010), European Commission (2010), and Graedel et al. (2015), for example. Nassar et al. (2015) assesses the degree to which metals are produced as joint products in the context of criticality.

of the subject is contained in Section 2. Section 3 follows with details of the estimation strategy and data. Elasticity estimates are presented in Section 4, and Section 5 describes the results' implications.

2 Mining Firm Behavior

The supply of metal from a given mine can be generally determined by the decisions made in two stages of production: mining and milling. At the mining stage, a producer chooses the quantity and quality of ore to extract from the ground. At the milling stage, a producer chooses the efficiency of processing that ore into a higher quality, concentrate product. The concentrate contains one or more metals and some waste. This concentrate is usually sold to a downstream refiner or smelter who and processes it into separated metal products. Because the refiner is often not directly tied to a single mine, this stage is excluded from our the analysis. A very simple model of metal produced from mining and milling is presented in Equation 1.

$$ContainedQuantity_i = OreMined * Grade_i * MillRecovery_i \tag{1}$$

$ContainedQuantity_i$ is the quantity of metal i contained in the mill concentrate, the product a mine typically sells downstream. $OreMined$ is the quantity of ore (metal + waste) extracted from the ground. $Grade_i$ is the percentage of metal i contained in the ore. Because mineral deposits are typically heterogeneous in quality, grade can vary depending on where the deposit is being mined. MillRecovery is the efficiency of the milling processes, measured as the percentage of metal i that is recovered from waste.

In this simple model, producers responding to higher prices by lowering grade can have a negative own-price elasticity of supply if ore and recovery are fixed. The primary empirical model in this paper holds ore fixed, and so it can be thought of as the elasticity of removed "in-situ" quantity (OreMined*Grade) with respect to price. Alternative specifications allow us to also hold the recovery rate fixed. For these specifications, a decrease in grade directly translates to a decrease in output.

The short-run relationship between a metal's price and its produced grade has been the subject of some debate over the last three decades, both in terms of optimality for profit maximization and empirical realities. A negative relationship between price and supplied metal quantity was first noted in the economics literature for the case of gold by Keynes (1936) and Paish (1938). This observation was followed by more sophisticated models on grade choice, each offering their own set of assumptions which make the negative relationship between price and grade optimal (or not).

Studies which have examined the issue of optimal grade choice in mining using Hotelling-style optimal control models have found conflicting results, both in terms of why firms might optimize by lowering grade and whether a negative relationship is optimal at all. Shinkuma (2000) proposes that the negative relationship of metal price to grade is due to "disorderliness"[4] in the composition of the deposit which impacts the mine's cost. Slade (1988), on the other hand, finds that the negative relationship is optimal when there is uncertainty in future prices. Similarly, Krautkraemer (1989) models a situation where negative own price response is optimal when low grade ore must be mined "now or never," price changes are unanticipated, and the mine life is endogenous. Cairns and Shinkuma (2003) approach the problem differently by creating a more general and complex model without a predetermined optimal response. The authors find an ambiguous relationship between price and grade depending on parametrization. Napier (1983) is the only identified paper that either finds or assumes mines increase grades when prices increase.

Empirical tests for the relationships between grade and price have relied on relatively small samples over a limited number of commodities. Farrow and Krautkraemer (1989) utilize a probit model and sample of 38 South African gold mines over eight years and 5 Idahoan silver, lead, and zinc mines over twelve years to test the relationship between price and grade. For the Idaho mines, variables are added to account for cross-price effects. The authors find a negative relationship between price and grade in South Africa. For the Idaho mines, they find a negative own-price relationship for silver, lead, and an insignificant negative result for zinc. They also find significant negative cross-price effects for changes in zinc price on silver and lead grades. Marsh (1983a) statistically tests the theory using a sample of 29 South African gold mines over seven years. In a similar analysis, Marsh (1983b) looks at gold and uranium mines with potential cross-price effects. Both Marsh (1983a) and Marsh (1983b) find evidence of the negative relationship between price and grade in the short run. The results found by Marsh (1983a), Marsh (1983b), and Farrow and Krautkraemer (1989) for South Africa may be due to a policy which essentially mandates such behavior for gold miners (Slade, 1988), but it is unclear when this policy was put into place. Finally, Shinkuma and Nishiyama (2000) use graphical analysis of copper deposits to test the theory proposed by Shinkuma (2000); results are mixed. In a sample of 51 mines, they find that nearly all "disorderly" deposits exhibit (weakly) the negative price-grade response, but some "orderly" deposits do as well.

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[4] In Shinkuma and Nishiyama (2000), deposits with little or no pattern in ore quality distribution are said to be disorderly. For copper, the authors consider porphyry copper and sedimentary deposits to be orderly, while skarn and volcanogenic massive sulfide deposits are disorderly.

3 Estimation Strategy and Data

Previous studies which have empirically examined the behavior of multi-product mining firms have not utilized flexible functional form analysis[5] typical of production studies and well suited for the analysis of multi-product firms. The benefit of using a flexible functional forms over reduced form analysis in this application is the interpretability and consistency of estimated elasticity values. It also avoids equation-by-equation OLS estimation which has been shown by Vinod (1968) to be biased for jointly-produced products.

We utilize a revenue function approach to estimate own and cross-price elasticities of supply. A revenue function is chosen over a profit function for several reasons. First, this analysis is output oriented, and so the responsiveness of firms to input prices is not directly of interest. Second, mining firms face a strict capacity constraint (typically the quantity of ore that can be milled in a given period). Assuming that mines seek to produce at this capacity, total input effort can be considered fixed. Additionally, if the transformation function is additively input-output separable then relative input prices have no effect on relative outputs and modeling them explicitly should have no theoretic impact on the result. Finally, a more practical consideration is the lack of public data on input costs for mining operations.

The empirical specification follows Kirkley and Strand (1988), who utilize a generalized Leontief revenue function for analysis of supply response in a fishery. The generalized Leontief function, first proposed by Diewert (1971), allows for estimation using levels rather than share data (as required by the translog function). As in Kirkley and Strand (1988), the revenue function takes a non-homothetic form to account for changes in relative grades as a deposit is mined down. The revenue function for each mine, k, takes the explicit form[6] shown in Equation 2.

$$R_k(P,Z) = \sum_i \sum_j \beta_{ij}(P_iP_j)^{1/2}Z_k + \sum_i \beta_i(P_iZ_k^2 + a_kP_i) \tag{2}$$

where P_i is the price[7] of the i^{th} metal product, Z_k is the composite input effort of the k^{th} mine, measured here as ore mined/milled, and a_k is an individual mine fixed effect.[8] β's are parameters to be estimated.

Applying Hotelling's Lemma results in the input compensated "in-situ" (Ore*Grade) supply functions for each metal, $Q_i(P,Z)$:

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[5] Fizaine (2013) and Mudd et al. (2013) both utilize co-integration analysis. Afflerbach et al. (2014) use OLS regression on monthly price data. The study by Livernois and Ryan (1989) is an exception. They utilize a variable profit function approach with an application in oil and gas.

[6] The time index has been removed for presentation simplicity.

[7] Prices are assumed to be exogenous. See Section 4.1 for a discussion.

[8] This fixed revenue effect could include effects such as smelter contracts which affect the prices the mine receives.

$$\frac{\partial R}{\partial P_i} = Q_{ik}(P,Z) = \beta_{ii} Z_k + \beta_i Z_k^2 + \sum_j \beta_{ij} (P_j/P_i)^{.5} Z_k + a_k \tag{3}$$

where $i \neq j$

Equation 3 is estimated as a seeming unrelated regression (SUR) model using the two step method. Coefficient symmetry, $\beta_{ij} = \beta_{ji}$, is imposed as a constraint on the estimation.

The data for the analysis are an unbalanced panel of 113 mines operating in thirty countries from the period 1991 to 2005. The data come from two sources. Mine level production data were downloaded freely from minecost.com in February 2015. The dataset has since been purchased by SNL Financial and is no longer freely available. Variables from minecost.com are tonnes of ore mined, average grade of metal i in ore mined, tonnes of ore milled, mill recovery rate of metal i, and quantity of metal i in produced concentrate. Metal price data come from the USGS series 140.[9]

Five metal commodities: gold (Au), silver (Ag), copper (Cu), lead (Pb), and zinc (Zn), were selected from a larger sample for the analysis. This sub-setting was chosen to maximize the number of metals produced from the same mines. No mines were included that produced only one product over the period. Eleven mines in the sub-sample of 113 produce all five products, while 34 produce just two.

Table 1 presents a summary of the revenue share of each of the five metals. The overall average shares across all 113 mines in the sample varies from around 12% for silver (produced by 99 mines) to nearly 60% for zinc (produced by 73 mines). Gold and copper, which are produced by roughly half the mines in the sample, account for close to 40% of revenue each, on average. However, in mines that produce lead/zinc, gold and copper account for much smaller shares of revenue, between 10%-15%.

Table 1: Average Revenue Share of Metals, Overall and in Mines Producing Selected Metals

		In mines which produce at least:				
	All Mines	Silver	Gold	Copper	Lead	Zinc
Number of Mines (#)	113	99	60	60	62	73
Silver Revenue (%)	12	12	12	11	13	11
Gold Revenue (%)	37	39	39	18	13	12
Copper Revenue (%)	37	32	46	39	8	16
Lead Revenue (%)	20	20	14	14	21	20
Zinc Revenue (%)	59	57	45	54	61	60

Note: Percentages should not necessary add to 100.

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[9] Series 140 prices are based on a weighted average price of US imports, representative of world prices, and provide a uniform basis of comparison. Indexed prices are plotted over the relevant time period in Figure B.1

4 Results

This section presents the coefficient estimates of Equation 3 and the resulting own and cross-price elasticities. The estimates point to three potentially surprising findings. The own-price estimates suggest that metals have a negative short-run supply curve. The positive and negative cross price results, paired with the negative own price results, suggests that metals are both complements and substitutes in supply. Finally, the large own and cross-price elasticity estimates for low revenue products silver and lead, and the negligible estimates for gold and copper, call into question a framework for thinking of joint mineral products in terms of main, co and by-products.

The regression estimates of Equation 3 are presented in Table B.1 of the Appendix. These results are difficult to interpret on their own, but worth noting is the significance of the coefficients associated with the input effort and the cross-price effects. For two metals, i and j, the price ratio term $(P_j/P_i)^{1/2}$ in Equation 3 is labeled j_i in the estimation results table. Aggregate input effort, Z, is labeled OreMinedkt (kt being thousand tonnes). The dependent variable is $Metal_i\_InSitu$ which is equal to $Grade_i * OreMinedkt$.

Own-price and cross-price elasticity values are calculated using Equations 4 and 5, respectively.

$$E_{ii} = \frac{\partial Q_i}{\partial P_i}\frac{P_i}{Q_i} = \frac{-Z}{2P_i^{1/2}Q_i}(\sum_j \beta_{ij}P_j^{1/2}) \tag{4}$$

$$E_{ij} = \frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{\beta_{ij}ZP_j^{1/2}}{2Q_iP_i^{-1/2}} \tag{5}$$

The elasticity results are presented in Table 2, and are evaluated at mean levels of Q, Z, P, and represent average effects. Standard errors of the regression are bootstrapped and calculated using the delta method as described in (Greene, 2012, p 1123) and implemented with the *R* package described in Jackson (2011).

Consistent with past theoretic and empirical work, own-price elasticities should be significant and negative. Further, if the conventional wisdom on jointly produced metals were correct, we would expect that main products and co-products should have a larger impact on their own and cross-product supply than by-products. By-product prices, by definition, should have no statistically significant effect on their own or other metal supply. Ways of identifying how important a given metal is to a mine's operation include the revenue or profit that a particular metal contributes, or some other method (Nassar et al., 2015). We form our expectations based on the average revenue estimates from Table 1. From these estimates, Zinc's nearly 60% average revenue share clearly makes it the most important metal, on average, for the mines in this sample. Gold and copper follow with 37% shares. Finally, lead and silver make up less than 20% of revenue on average. From these numbers, we expect silver and lead production should largely be driven by the prices of the other metals, while zinc prices should drive production of the other metals.

Table 2: Input-Compensated Own and Cross-Price Elasticities, Mining Stage

	P Gold	P Silver	P Copper	P Lead	P Zinc
Q Gold	0.097	-0.04	0.507 ***	-0.184	-0.381
	(0.466)	(0.044)	(0.082)	(0.123)	(0.443)
Q Silver	-0.117	-6.916 ***	0.46 *	0.466	6.107 ***
	(0.13)	(1.148)	(0.22)	(0.429)	(1.124)
Q Copper	0.254 ***	0.079 *	0.112	-0.407 ***	-0.038
	(0.041)	(0.038)	(0.21)	(0.103)	(0.177)
Q Lead	-0.511	0.441	-2.255 ***	-2.987 ***	5.312 ***
	(0.341)	(0.406)	(0.57)	(0.519)	(0.627)
Q Zinc	-0.322	1.761 ***	-0.065	1.618 ***	-2.992 ***
	(0.375)	(0.324)	(0.298)	(0.191)	(0.285)

P's are prices, Q's are quantities.
Elasticities are presented at their average values and calculated using Equations 4 and 5.
Bootstrapped standard errors are in parentheses and estimated using the delta method.
$*p < 0.05$, $**p < .01$, $***p < .001$

Own-price elasticities are represented along the main diagonals of Table 2, and cross-price elasticities on the off-diagonals. Silver, lead, and zinc all exhibit significant and negative own-price elasticity, consistent with the theory discussed in Section 2 that firms lower grade in response to higher prices. However, these results are inconsistent with our expectations that silver and lead should be less price responsive than other metals. Gold and copper's prices are estimated to have no statistically significant effect on their produced grade. This result is also puzzling because gold and copper's moderate revenue contribution would intuitively indicate that they should be produced as price sensitive co-products.

The significant (positive or negative) results for the cross-price elasticities indicate an interdependence in the supply of these metals. Of the possible cross-price effects, half are statistically significant at the 95, 99 or 99.9% level. As expected, zinc's price has a large effect on the production of silver and lead. A 1% increase in zinc prices result in a 6.1% increase in mined in-situ silver quantities and a 5.3% increase in lead. The prices of gold and copper have relatively little impact on the production of other metals. While their cross-elasticity values are smaller in magnitude, silver and lead again defy the expectations that their prices should not drive the production of other metals. For example, 1% increase in silver prices leads to a 0.1% increase in copper production and a 1.7% increase in zinc production. These results for silver and lead, as well as the own-price results for gold and copper, call into question the conventional wisdom on co-product and by-product behavior.

Model misspecification and data issues, further discussed in Appendix 4.1, are two potential reasons conventional wisdom on the nature of by-product/co-product/main product supply does not seem to hold. However, our results are not readily invalidated by alternative model specifications (See Section A of Appendix). Defining our elasticity expectations based on revenue share, as opposed to profit contribution (or some other measure), is another possible flaw. Revenue share therefore may not be a useful measure of a metal's importance to a mine.

The results point to a potential need for modified understanding of multi-product mineral supply. Only a handful of papers have considered the case of a multi-product mine producing from a heterogeneous resource, but this situation represents a significant share of metal supply. Considering that a heterogeneous resource and choice of grade can lead to a negative short-run supply response, it is potentially unsurprising that this more complex situation could also lead to equally complex response by multi-product firms, beyond more simple frameworks of by-products and main-products.

Cross-product relationships might be better defined in terms of substitute and complement relationships. These definitions may more appropriately reflect geologic features of a deposit like the relative distribution of metal grades across the resource. For metals with negative own price elasticity, a negative cross-price elasticity indicates a complement and a positive cross-price indicates a substitute (the reverse is true when a metal has a positive own-price relationship). For example, when the price of lead increases, mines shift production toward lower in-situ quantities of lead and copper, and higher grades of zinc. This makes lead and copper complements, and lead and zinc substitutes. The relationships become more ambiguous for gold and copper, however, because of their negligible own-price effects. These relationships are summarized in Table 3.

Table 3: Estimated Metal Substitute and Complement Relationships

Substitutes in Output	Complements in Output
Silver & Copper	Lead & Copper
Silver & Zinc	Gold & Copper*
Lead & Zinc	

*Assumes copper & gold have a positive (non-zero) own-price effect

While the own-price results are consistent with the limited past empirical work, the cross-price results are generally reversed. Farrow and Krautkraemer (1989) find a price increase in zinc results in decreased grades of silver and lead; our results show the opposite effect. Similarly, Marsh (1983b) finds a negative relationship between gold price and uranium grades in contrast to our cross-price results which are 4-to-1 positive.

Marsh (1983b) does not include uranium prices in his analysis. If uranium exhibits a negative price-grade relationship and uranium prices are positively correlated with gold prices, the gold-uranium cross-elasticity estimate is negatively biased. Farrow and Krautkraemer (1989) include a full set of prices in their model which makes it harder to explain the discrepancy. It is possible that the results from Marsh (1983b) and Farrow and Krautkraemer (1989) are valid for a more limited sampling of mines. It is also possible that the flexible functional form analysis reveals a more complete, and complex, set of multi-product price responses.

4.1 Robustness of Findings

The empirical model has assumed that firms are price takers and that metal prices are exogenous. However, if this assumption is invalid and prices are endogenous then coefficient estimates will be biased by system simultaneity and price responses will be overestimated. Because the sample of mines includes a notable percentage of world production (around 30%) for each of the five metals, it is difficult to argue that overall production is inconsequential. In alternative reduced-form specifications for the empirical model, a two-stage least squares estimator was considered and tested for the relationship between price and grade. From the results, price endogeneity is not found to be a driver of the results. See Appendix A.1.

The finding that mines do not seem to follow the conventional framework for joint-products is more formally tested using an alternative model specification in Appendix A.2. These quantitative tests align with the Results in Section 4 that by-products and main products, as defined by revenue contribution, do not necessary differ in their own and cross-price responsiveness.

The results are also not explained by the type of mining operation (open pit versus underground). Despite gold and copper being produced in relatively higher frequency from open pit mines, as shown in Table 4, this factor alone does not seem to drive our results. When more formally tested by regressing grade on an interacted dummy variable and controls, we find no statistical difference between open pit and underground mines for any metal. The output of this regression model is presented in Table B.6.

Table 4: Percentage of Observations from Open Pit and Underground Mines

	Silver	Gold	Copper	Lead	Zinc
Open Pit (%)	20.00	30.00	23.00	10.00	10.00
Underground (%)	53.00	42.00	42.00	68.00	68.00

5 Conclusions

This study finds significant cross-price elasticity estimates for many of the metal-pairs assessed. While the estimates themselves may be of interest to analysts of these particular markets, more generally they should call attention to the fact that metal supply should not be considered in isolation. The geologic processes which have concentrated these materials together into economically exploitable deposits have also critically linked their supply. The complexity of these geologic processes leads to equally complex responses when economic conditions change.

As this paper has shown, metals can behave as substitutes and complements in the supply of other metals. These relationships are likely driven by the heterogeneous nature of mineral deposits which have grade distributions that vary over space. While it may be tempting to consider low revenue products unimportant to the mine and label them by-products, a more useful label appears to be complement or substitute. For instance, mines seem to respond to the prices of silver and lead, neither of which contribute more than 20% to revenue on average. However, production of copper and gold, both of which contribute more notably to revenue (almost 40%), do not generally respond to changes in their own price. Another important finding of this study is the negative own price effect which can lead to a negative overall supply if recovery rates and ore tonnage are fixed.

The findings of this paper highlight how interdependence in metal supply can defy certain expectations based on conventional wisdom. Increased price for a given metal can drive increased production of joint-products but a decrease in supply of that metal in the short run. As rapid deployment of emerging technologies drive increased demand for certain metals, price increases can lead to a short-run reduction in supply in those metals as capacity-constrained producers move to exploit lower grades. While new capacity will eventually come online to meet demand, end users could face higher prices for many years.

References

Afflerbach, P., G. Fridgen, R. Keller, A. W. Rathgeber, and F. Strobel (2014). The by-product effect on metal markets–new insights to the price behavior of minor metals. *Resources Policy 42*, 35–44. DOI: `10.1016/j.resourpol.2014.08.003`.

Bauer, D., D. Diamond, J. Li, D. Sandalow, M. M, and P. Telleen (2010). US Department of Energy Critical Materials Strategy. Available at: `http://energy.gov/sites/prod/files/edg/news/documents/criticalmaterialsstrategy.pdf`.

Cairns, R. D. and T. Shinkuma (2003). The choice of the cutoff grade in mining. *Resources Policy 29*(3), 75–81. DOI: `10.1016/j.resourpol.2004.06.002`.

Diewert, W. E. (1971). An application of the shephard duality theorem: A generalized leontief production function. *The Journal of Political Economy*, 481–507.

European Commission (2010). Critical raw materials for the EU. *Report of the Ad-hoc Working Group on defining critical raw materials*. Available at: `http://ec.europa.eu/growth/sectors/raw-materials/specific-interest/critical/index_en.htm`.

Farrow, S. and J. A. Krautkraemer (1989). Extraction at the intensive margin: metal supply and grade selection in response to anticipated and unanticipated price changes. *Resources and Energy 11*(1), 1–21. DOI: `10.1016/0165-0572(89)90002-9`.

Fizaine, F. (2013). Byproduct production of minor metals: Threat or opportunity for the development of clean technologies? the pv sector as an illustration. *Resources Policy 38*(3), 373–383. DOI: `10.1016/j.resourpol.2013.05.002`.

Graedel, T., E. Harper, N. Nassar, and B. K. Reck (2015). On the materials basis of modern society. *Proceedings of the National Academy of Sciences 112*(20), 6295–6300. DOI: `10.1073/pnas.1312752110`.

Greene, W. H. (2012). *Econometric Analysis* (7th ed.). Pearson Education.

Jackson, C. H. (2011). Multi-state models for panel data: The msm package for R. *Journal of Statistical Software 38*(8), 1–29. `http://www.jstatsoft.org/v38/i08/`.

Jensen, C. L. (2002). Applications of dual theory in fisheries: A survey. *Marine Resource Economics 17*(4), pp. 309–334. `http://www.jstor.org/stable/42629372`.

Keynes, J. (1936). The supply of gold. *The Economic Journal*, 412–418. DOI: `10.2307/2224879`.

Kirkley, J. E. and I. E. Strand (1988). The technology and management of multi-species fisheries. *Applied Economics 20*(10), 1279–1292. DOI: `10.1080/00036848800000100`.

Krautkraemer, J. A. (1989). Price expectations, ore quality selection, and the supply of a nonrenewable resource. *Journal of Environmental Economics and Management 16*(3), 253–267. DOI: `10.1016/0095-0696(89)90013-2`.

Livernois, J. R. and D. L. Ryan (1989). Testing for non-jointness in oil and gas exploration: a variable profit function approach. *International Economic Review*, 479–504. DOI: `10.2307/2526658`.

Marsh, J. B. (1983a). Keynes on the supply of gold: A statistical test. *Eastern Economic Journal*, 7–12. `http://www.jstor.org/stable/40325019`.

Marsh, J. B. (1983b). Some evidence of price and cross-elasticities of mineral supply: The case of gold and uranium. *Resources and Energy 5*(2), 105–113. DOI: `10.1016/0165-0572(83)90008-7`.

Mudd, G., Z. Weng, S. Jowitt, I. Turnbull, and T. Graedel (2013). Quantifying the recoverable resources of by-product metals: The case of cobalt. *Ore Geology Reviews 55*, 87–98. DOI: `10.1016/j.oregeorev.2013.04.010`.

Napier, J. (1983). The effect of cost and price fluctuations on the optimum choice of mine cut-off grade. *Journal of the South African Institute of Mining and Metallurgy 83*(6), 117–125. Available at: `http://www.saimm.co.za/Journal/v083n06p117.pdf`.

Nassar, N., T. Graedel, and E. Harper (2015). By-product metals are technologically essential but have problematic supply. *Science Advances 1*(3), e1400180. `DOI:10.1126/sciadv.1400180`.

Paish, F. W. (1938). Causes of changes in gold supply. *Economica*, 379–409. DOI: `10.2307/2548993`.

Pindyck, R. S. (1982). Jointly produced exhaustible resources. *Journal of Environmental Economics and Management 9*(4), 291–303. DOI: `10.1016/0095-0696(82)90023-7`.

Shinkuma, T. (2000). A generalization of the cairns–krautkraemer model and the optimality of the mining rule. *Resource and energy economics 22*(2), 147–160. DOI: `10.1016/S0928-7655(99)00020-2`.

Shinkuma, T. and T. Nishiyama (2000). The grade selection rule of the metal mines; an empirical study on copper mines. *Resources Policy 26*(1), 31–38. DOI: `10.1016/S0301-4207(00)00014-3`.

Slade, M. E. (1988). Grade selection under uncertainty: least cost last and other anomalies. *Journal of Environmental Economics and Management 15*(2), 189–205. DOI: `10.1016/0095-0696(88)90018-6`.

Tilton, J. E. (1990). Economics of the mineral industries. In H. Hartman (Ed.), *SME Mining Engineering Handbook*. Society for Mining, Metallurgy, and Exploration, Inc.

Vinod, H. D. (1968). Econometrics of joint production. *Econometrica: Journal of the Econometric Society*, 322–336. DOI: `10.2307/1907492`.

Appendix A Robustness of Findings and Alternative Models

This appendix will detail several extensions that were used in order to validate results and tests hypotheses. An alternative model was used in the cases where the primary model did not have necessary flexibility in order to add extensions. These alternative models are based on the empirical model in Equation A.1.

$$Grade_{itm} = (\sum_{j \neq i} Grade_{jtm} + \sum_{i} Price_{it}) * PDum_{im} + Ore_{tm} + \sum^{t} Ore_{mt} + \alpha_m \tag{A.1}$$

where i and j are the set of metals, silver, gold, copper, lead, and zinc. $Grade_{itm}$ measures the percent of metal i in ore mined in time t by producer m. $PDum_{im}$ indicates if a particular metal is produced at a given mine, m. $PDum_i$ takes a value of 1 if a metal, i is produced at any time in the sample period and 0 otherwise. In regression output tables this dummy effect is inticated by a _D. Ore_{mt} measures contemporaneous annual ore production from mine m at time t. The sum of Ore to the current period represents cumulative production. α is a mine inividual fixed effect. The estimates from Equation A.1 are presented in Table B.2. In this "base case" model, only lead's own price-grade relationship is significant (and negative). Additonally, three cross-price effects Ag-Au, Pb-Au, and Zn-Pb are signficant at the 99.9, 95, and 99.9% levels, respectively. This model will form the basis for comparison of the alternative specifications.

A.1 Price Endogeneity and Instrumental Variables

As discussed in Section 4.1, one potential concern about the primary model's specification is that prices are assumed exogenous. While this is the same assumption of all past studies on the price/grade relationship, the potential for prices to be simultaneously determined with grades warrants testings. The primary SUR model does not lend itself to an instrumental variable correction, even through three stage least-squares, because not all prices are relevant to all mines. Because the set of instruments (via dummying) will be different for each metal's price, the 2-stage regression is invalid. We instead fit the first and second stage results separately resulting in an unbiased estimate of the prices' coefficients and correct standard errors via bootstrapping. The results of the IV estimation are presented in Table B.3. After instrumenting, lead maintains a significant and negative own price-grade effect. All other own price effects are insignificant, as in the base case results in Table B.2. Also, the three cross price effects that were significant in the base case model retain their significance and signs. The magnitude of the coefficient estimates is also similar between the two models. The similarity provides some confidence that endogeneity is not introducing major bias into the results of the primary or alternative models.

A.2 Formal Tests of Revenue Effects

In this section we formally test the finding in the primary model that a metal's revenue contribution is a poor indicator of its own and cross price responsiveness. Utilizing the alternative model in Equation A.1, and adding time fixed effects (η_t) and milling recovery rate ($Recov_{it}$) control variables, we estimate the model in Equation A.2.

$$Grade_{itm} = (\sum_{j \neq i} Grade_{jtm} + \sum_{i} Price_{it} * RevDum_{itm}) * PDum_{im} + Recov_{it} + Ore_{tm} + \sum^{t} Ore_{mt} + \eta_t + \alpha_m \tag{A.2}$$

The variable of interest in Equation A.2 is $RevDum_{itm}$, a dummy variable which indicates a metal's revenue contribution. A metal is considered a by-product if its contribution to total revenue is less than 10%, considered a co-product between 10 and 75% and a main product for revenue greater than 75%. These bounds are arbitrary, but a useful starting place for the analysis. Choosing different percentage revenue cut-offs (even down to 1% revenue for by-products) does not meaningfully change the results. The estimated coefficients of Equation A.2 are presented in Tables B.4 and B.5.

The expectation is that main products should have large impacts on their own and other-metal supply, while by-product prices should have no effect on their own supply or the supply of other materials. This is examined using F tests of the estimates in Tables B.4 and B.5 with the test specifications and expectations in Table A.1. Conventional wisdom suggests that the byproduct dummy interacted with price should not have a statistically significant effect on the grade of the by-product or other metals. We expect that the main product dummy interacted with price will have statistically significant effect on the grade mined both of the main product and other metals. Finally, we use a Wald test to determine if the main product effect is statistically different than the by product-effect, which it intuitively should be. The results of the three tests are summarized for each of the 5 metals in Tables A.2 and A.3.

Table A.1: Revenue Interaction Tests and Expectations

Test Name	Specification	Expectation
By-product	H_0: $(ByProduct\_RevDum_{itm} + 1) * Price_{it} = 0$	Not Reject
Main Product	H_0: $(MainProduct\_RevDum_{itm} + 1) * Price_{it} = 0$	Reject
Differnce	H_0: $(MainProduct\_RevDum_{itm} + 1) * Price_{it} =$ $(ByProduct\_RevDum_{itm} + 1) * Price_{it}$	Reject

Comparing the test outcomes in Table A.2 to the expectations in Table A.1, we see somewhat mixed results. When produced as by-products, silver, gold, copper, and lead are not own-price responsive (as expected), but zinc is. Contrary to expectations gold, copper, and lead are not own-price responsive when

Table A.2: Test Results for Revenue Interactions, Own Grade-Price Results

Revenue Effect	Silver Grade	Gold Grade	Copper Grade	Lead Grade	Zinc Grade
Byproduct Effect	0	0	0	0	-
Main product Effect	-	0	0	0	-
Effects Different	Yes	No	No	Yes	No

Result of 0 indicates no significant ($\alpha = 90\%$) effect.

+/- denote direction of significant effect.

Yes/No indicates whether main product effect is statistically different than by-product effect.

Table A.3: Test Results for Revenue Interactions, Cross Grade-Price Results

	Silver Grade	Gold Grade	Copper Grade	Lead Grade	Zinc Grade
Byproduct Price Effect of	0	Pb	Pb, Zn	Ag, Au, Cu, Zn	0
Main product Price Effect of	0	0	Ag	0	Au

Result of 0 indicates no significant ($\alpha = 90\%$) effects.

Chemical symbols denote the relevant cross-prices.

produced as main products. We also expect the main product effect to be statistically different than the by-product effect, but our results show this is only the case for silver and lead. Of the 15 own-price tests, roughly half (8) conform to our expectations about behavior. The results for gold and copper are particularly notable because they echo the findings of the primary model.

The cross price effects are also interesting. While gold and copper are not responsive to their own prices, gold is responsive to the price of by-product lead, and copper is responsive to the prices of by-product lead and zinc. Lead is responsive to prices of all other metals when they are produced as by-products. Also surprising, there only two significant main product effects, far fewer than the seven by-product effects. A price increase in main-product silver drives an increase in mined copper grades, and a price increase in main-product gold results in an increase of mined grades of zinc. Of the 40 total cross-price tests, only 15 conform to our exceptions about behavior.

These tests of the the revenue interaction terms provides quantitative confirmation of the qualitative conclusion in Section 4 that conventional wisdom on the short-run behavior of by-products and main products may need to be revisited.

Appendix B Additional Tables and Figures

Table B.1: SUR Estimation Results by Metal at Mining Stage

	$Metal_i\_InSitu$	SEs
	Gold	
OreMinedkt	-0.0000*	(0.0000)
OreMinedkt*OreMinedkt	0.0000***	(0.0000)
Ag_Au*OreMinedkt	-0.0001	(0.0001)
Cu_Au*OreMinedkt	0.0058***	(0.0009)
Pb_Au*OreMinedkt	-0.0033	(0.0021)
Zn_Au*OreMinedkt	-0.0062	(0.0071)
	Silver	
OreMinedkt	-0.0011	(0.0010)
OreMinedkt*OreMinedkt	0.0000	(0.0000)
Au_Ag*OreMinedkt	-0.0001	(0.0001)
Cu_Ag*OreMinedkt	0.0148*	(0.0068)
Pb_Ag*OreMinedkt	0.0232	(0.0209)
Zn_Ag*OreMinedkt	0.2782***	(0.0529)
	Copper	
OreMinedkt	0.3380***	(0.0889)
OreMinedkt*OreMinedkt	-0.0000	(0.0000)
Au_Cu*OreMinedkt	0.0058***	(0.0009)
Ag_Cu*OreMinedkt	0.0148*	(0.0068)
Pb_Cu*OreMinedkt	-1.0020***	(0.2403)
Zn_Cu*OreMinedkt	-0.0863	(0.4029)
	Lead	
OreMinedkt	-0.0004	(0.0011)
OreMinedkt*OreMinedkt	0.0000	(0.0000)
Au_Pb*OreMinedkt	-0.0033	(0.0021)
Ag_Pb*OreMinedkt	0.0232	(0.0209)
Cu_Pb*OreMinedkt	-1.0020***	(0.2403)
Zn_Pb*OreMinedkt	3.3356***	(0.3938)
	Zinc	
OreMinedkt	0.0208***	(0.0040)
OreMinedkt*OreMinedkt	-0.0000***	(0.0000)
Au_Zn*OreMinedkt	-0.0062	(0.0071)
Ag_Zn*OreMinedkt	0.2782***	(0.0529)
Cu_Zn*OreMinedkt	-0.0863	(0.4029)
Pb_Zn*OreMinedkt	3.3356***	(0.3938)
N	1199	
r2	0.9067	
chi2	7972.3359	
p	0.0000	

$Metal_i\_InSitu$ is Q_i, OreMinedkt is Z, and $Metal_j\_Metal_i$ is the label for $(P_j/P_i)^{1/2}$ in Equation 3.

Bootstrapped standard errors in parentheses

* $p < 0.05$, ** $p < .01$, *** $p < .001$

Table B.2: Base Case for Alternative Specification

	(1) Ag_Grade_D	(2) Au_Grade_D	(3) Cu_Grade_D	(4) Pb_Grade_D	(5) Zn_Grade_D
Ag_nP_D	-4.568e-09	-1.526e-09***	2.531e-07	-1.174e-07	7.576e-07
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Au_nP_D	-1.725e-10	1.040e-11	-7.334e-09	-1.500e-08	6.421e-09
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Cu_nP_D	2.295e-07	1.616e-08	-3.241e-05	2.916e-05	-2.049e-05
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Pb_nP_D	-1.281e-07	1.942e-07*	-3.891e-05	-1.042e-03***	-3.218e-04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Zn_nP_D	-5.019e-07	3.188e-08	-1.018e-05	4.699e-04***	-2.159e-05
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	1169	1169	1169	1169	1169
r2	1.862e-01	1.954e-01	2.153e-01	2.155e-01	2.918e-01
chi2					
p	1.088e-213	5.192e-223	9.278e-243	5.245e-243	2.566e-313

Standard errors in parentheses

* $p < 0.05$, ** $p < .01$, *** $p < .001$

Control variables: Grades of other metals, cumulative production, and current production.

Table B.3: Alternative Specification - 2SLS Estimates

	(1) Ag_Grade_D	(2) Au_Grade_D	(3) Cu_Grade_D	(4) Pb_Grade_D	(5) Zn_Grade_D
Ag_nP_D_Inst	1.482e-09	-9.308e-10*	4.732e-07	-3.217e-07	1.037e-06
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Au_nP_D_Inst	-1.596e-10	6.146e-12	-4.930e-09	-1.474e-08	7.580e-09
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Cu_nP_D_Inst	9.890e-08	9.797e-09	-3.967e-05	2.379e-05	-2.743e-05
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Pb_nP_D_Inst	-1.219e-06	1.262e-07*	-9.192e-06	-1.044e-03***	-1.908e-04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Zn_nP_D_Inst	-4.653e-07	2.838e-08	-8.101e-05	5.541e-04***	-1.413e-04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	1122	1122	1122	1122	1122
r2	3.687e-01	3.753e-01	2.172e-01	2.070e-01	2.863e-01
chi2	.	.	9.466e+01	4.576e+02	1.059e+02
p	.	.	2.021e-15	3.457e-91	1.195e-17

Bootstrapped standard errors in parentheses.

* $p < 0.05$, ** $p < .01$, *** $p < .001$

Control variables: grades of other metals, cumulative production, and current production. Instrumental variables are: world GDP, population density of East Asia, arms imports to middle income countries, global steel production, production and stock of vehicles, the US CPI-U, US inflation, US inflation volatility, and a trade-weighted measure of the US foreign exchange rate.

Table B.4: Alternative Specification - Revenue Interactions

	Ag_Grade	Au_Grade	Cu_Grade	Pb_Grade	Zn_Grade
0.Ag_RevDum	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Ag_RevDum	-1.589e-04	9.102e-05	-2.136e+00*	-7.333e-01	-4.220e+00**
	(0.009)	(0.001)	(1.052)	(0.884)	(1.392)
2.Ag_RevDum	7.347e-03	-2.111e-04	-2.244e+00*	3.477e-01	-5.044e+00***
	(0.009)	(0.001)	(1.051)	(0.924)	(1.444)
3.Ag_RevDum	2.491e-02	-6.205e-04	-7.709e+00***	-1.834e+00	-8.936e+00**
	(0.013)	(0.002)	(1.483)	(1.722)	(2.721)
Ag_nP_D	-4.574e-08	2.888e-11	-2.711e-05***	-1.651e-05**	-2.368e-05*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
0.Ag_RevDum*Ag_nP_D	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Ag_RevDum*Ag_nP_D	1.067e-08	-4.401e-10	2.815e-05***	9.664e-06	2.499e-05**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
2.Ag_RevDum*Ag_nP_D	-2.609e-08	1.045e-09	2.860e-05***	4.147e-06	2.910e-05***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3.Ag_RevDum*Ag_nP_D	-6.312e-08	1.885e-09	3.311e-05***	1.132e-05	3.204e-05*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
0.Au_RevDum	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Au_RevDum	-1.553e-02	4.724e-04	5.669e-01	9.277e-01	-2.821e+00
	(0.017)	(0.001)	(0.908)	(1.557)	(2.451)
2.Au_RevDum	-1.681e-02	5.661e-04	-9.187e-02	-6.294e-01	-4.072e+00
	(0.017)	(0.001)	(0.924)	(1.600)	(2.513)
3.Au_RevDum	-1.426e-02	6.846e-05	-7.288e-02		-1.881e+01***
	(0.018)	(0.001)	(1.046)		(5.279)
Au_nP_D	-1.285e-09	5.753e-11	6.460e-08	2.658e-08	-3.590e-07
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
0.Au_RevDum*Au_nP_D	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Au_RevDum*Au_nP_D	1.250e-09	-4.040e-11	-4.528e-08	-1.408e-07	3.639e-07
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
2.Au_RevDum*Au_nP_D	1.398e-09	-5.033e-11	6.243e-09	-7.474e-09	4.109e-07
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3.Au_RevDum*Au_nP_D	6.749e-10	-3.728e-12	-4.205e-08		1.745e-06***
	(0.000)	(0.000)	(0.000)		(0.000)
0.Cu_RevDum	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Cu_RevDum	3.438e-03	-8.695e-05	1.705e-01	1.307e-01	-1.061e+00
	(0.004)	(0.001)	(0.245)	(0.403)	(0.635)
2.Cu_RevDum	1.702e-03	-1.778e-04	8.333e-01**	2.603e-01	-1.900e+00**
	(0.005)	(0.001)	(0.266)	(0.467)	(0.717)
3.Cu_RevDum	2.759e-03	-2.524e-04	6.184e-01*	3.830e-01	-1.110e+00
	(0.006)	(0.001)	(0.290)	(2.102)	(1.955)
Cu_nP_D	1.675e-06	-5.345e-08	-2.156e-04	1.803e-04	-3.985e-04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
0.Cu_RevDum*Cu_nP_D	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Cu_RevDum*Cu_nP_D	-1.570e-06	1.951e-08	1.372e-04	-2.376e-05	2.960e-04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
2.Cu_RevDum*Cu_nP_D	-1.626e-06	1.185e-07	-5.272e-05	-1.861e-04	5.357e-04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3.Cu_RevDum*Cu_nP_D	-1.925e-06	1.237e-07	7.957e-05	9.642e-04	1.308e-04
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)

Standard errors in parentheses

* $p < 0.05$, ** $p < .01$, *** $p < .001$

Table B.5: Alternative Specification - Revenue Interactions (Continued)

	Ag_Grade	Au_Grade	Cu_Grade	Pb_Grade	Zn_Grade
0.Pb_RevDum	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Pb_RevDum	6.299e-03	-1.253e-03	-1.191e+01***	-4.468e+00	-3.486e+00
	(0.022)	(0.001)	(1.288)	(3.343)	(3.967)
2.Pb_RevDum	6.580e-03	-1.285e-03	-1.141e+01***	-3.490e+00	-2.798e+00
	(0.022)	(0.001)	(1.289)	(3.349)	(3.957)
3.Pb_RevDum	9.383e-03			-1.668e+00	-5.448e+00
	(0.024)			(3.470)	(4.221)
Pb_nP_D	4.648e-06	-2.862e-07	-8.955e-03***	-3.660e-03	-4.516e-03
	(0.000)	(0.000)	(0.001)	(0.003)	(0.004)
0.Pb_RevDum*Pb_nP_D	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Pb_RevDum*Pb_nP_D	-7.927e-06	1.068e-06	9.600e-03***	3.865e-03	4.313e-03
	(0.000)	(0.000)	(0.001)	(0.003)	(0.004)
2.Pb_RevDum*Pb_nP_D	-7.769e-06	1.116e-06	9.109e-03***	3.351e-03	3.652e-03
	(0.000)	(0.000)	(0.001)	(0.003)	(0.004)
3.Pb_RevDum*Pb_nP_D	-1.183e-05			2.159e-03	6.238e-03
	(0.000)			(0.003)	(0.004)
0.Zn_RevDum	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Zn_RevDum	3.376e-03	-1.012e-04	5.265e+00***	0.000e+00	9.936e+00***
	(0.018)	(0.001)	(1.136)	(.)	(2.966)
2.Zn_RevDum	-1.600e-03	-1.297e-04	4.190e+00***	2.414e+00**	9.923e+00***
	(0.017)	(0.001)	(0.925)	(0.741)	(2.767)
3.Zn_RevDum	-2.377e-03	-6.310e-04	4.468e+00***	2.112e+00**	1.054e+01***
	(0.017)	(0.001)	(0.957)	(0.778)	(2.771)
Zn_nP_D	3.399e-07	-1.479e-07	3.105e-03***	6.549e-04	3.492e-03
	(0.000)	(0.000)	(0.001)	(0.002)	(0.003)
0.Zn_RevDum*Zn_nP_D	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
1.Zn_RevDum*Zn_nP_D	3.040e-06	-1.753e-07	-4.333e-03***	2.885e-03	-4.539e-03*
	(0.000)	(0.000)	(0.001)	(0.002)	(0.002)
2.Zn_RevDum*Zn_nP_D	1.639e-06	-1.179e-07	-3.057e-03***	1.433e-03	-4.226e-03*
	(0.000)	(0.000)	(0.001)	(0.002)	(0.002)
3.Zn_RevDum*Zn_nP_D	1.368e-06	1.227e-07	-3.382e-03***	1.072e-03	-3.728e-03
	(0.000)	(0.000)	(0.001)	(0.002)	(0.002)
N	1015	574	605	692	770
r2	2.417e-01	2.908e-01	6.753e-01	4.393e-01	5.022e-01
chi2					
p	2.603e-51	2.989e-21	2.194e-123	6.359e-72	2.795e-105

Standard errors in parentheses

* $p < 0.05$, ** $p < .01$, *** $p < .001$

By-product dummy value is 1, co-product 2, and main product 3.

Control variables: Grades of other metals, cumulative production, and current production.

Table B.6: Alternative Specification - Mine Type Interaction

	Ag_Grade_D	Au_Grade_D	Cu_Grade_D	Pb_Grade_D	Zn_Grade_D
UGDum	0.000e+00	0.000e+00	0.000e+00	0.000e+00	0.000e+00
	(.)	(.)	(.)	(.)	(.)
Ag_nP_D	-6.887e-08	2.554e-09	2.196e-06	-7.631e-06	3.788e-06
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
UGDum*Ag_nP_D	-1.505e-08	-3.405e-09	1.525e-07	-2.506e-06	-2.772e-07
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Au_nP_D	-2.748e-10	-6.838e-12	4.135e-08	-2.348e-08	-6.699e-08
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
UGDum*Au_nP_D	6.765e-11	8.384e-12	-2.088e-08	-8.036e-09	-1.495e-08
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Cu_nP_D	1.287e-06	-5.322e-08	-9.249e-05	2.245e-04	1.726e-04
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
UGDum*Cu_nP_D	-6.612e-07	6.532e-08	-8.694e-06	-1.815e-04	-5.524e-05
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Pb_nP_D	-5.056e-07	3.343e-07	-5.823e-04	4.089e-04	5.690e-05
	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
UGDum*Pb_nP_D	8.717e-07	5.026e-07	5.191e-04	-4.871e-04	2.397e-04
	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
Zn_nP_D	3.337e-06	-4.323e-07	4.920e-04	2.140e-03	-3.170e-03
	(0.000)	(0.000)	(0.001)	(0.002)	(0.002)
UGDum*Zn_nP_D	-1.423e-06	1.369e-07	-6.443e-04	3.488e-04	1.588e-04
	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
N	1015	574	605	692	770
r2	2.157e-01	2.941e-01	4.007e-01	3.088e-01	3.986e-01
chi2					
p	1.712e-70	8.723e-37	3.140e-67	5.361e-58	9.607e-104

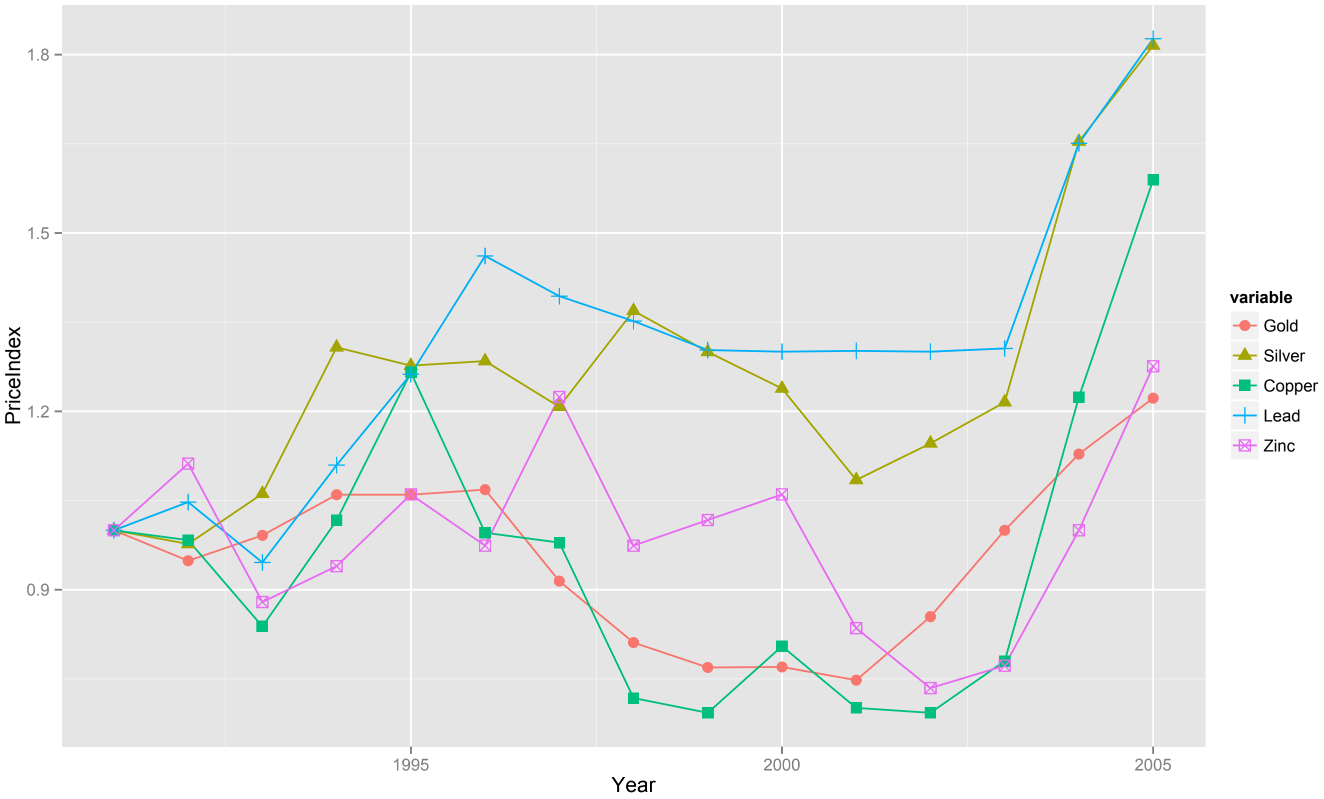
Standard errors in parentheses

* $p < 0.05$, ** $p < .01$, *** $p < .001$

UGDum take a value of 1 for underground Mines and 0 otherwise.

Controls variables: Grades of other metals, cumulative production, and current production.

Figure B.1: Selected Metal Prices from 1991-2005, Index 1991=1



Source: USGS series 140. Weighted Average US Import prices.