JAX FDM

A differentiable framework for constrained form-finding

Rafael Pastrana, Sigrid Adriaenssens

https://github.com/arpastrana/jax_fdm

Princeton University
School of Architecture

FORM
FINDING
Lab

Schlaich Bergermann, et al. (2018). Trumpf pedestrian footbridge. Ditzingen, Germany

Skidmore, Owens and Merill, et al. (2021). The roof of the Moynihan train hall. New York City, New York

Form-finding is shape optimization

A form-finding method computes a shape in static equilibrium

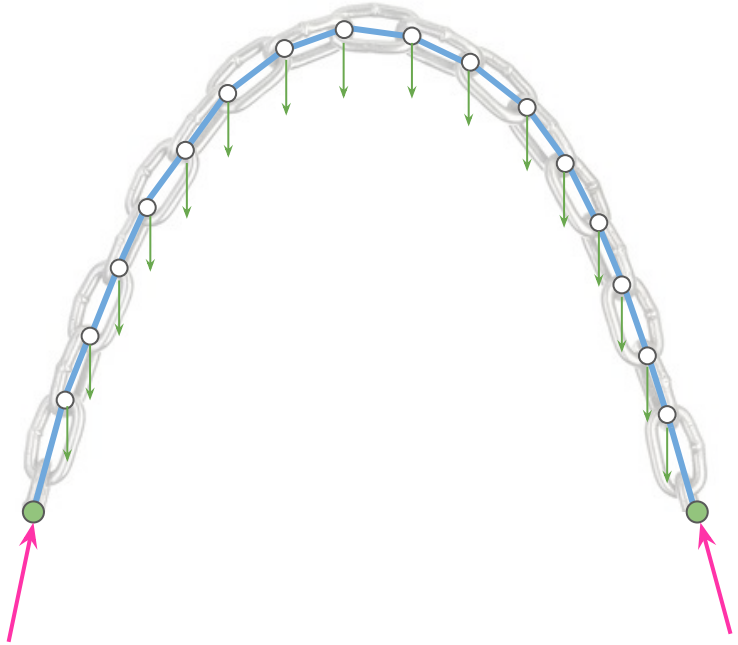
Tension-only

Compression-only

Photo credit: Adobe stock

Form-finding is shape optimization

A form-finding method computes a shape in static equilibrium



Form-finding is shape optimization

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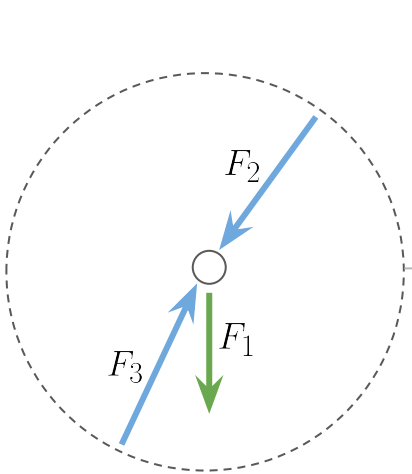
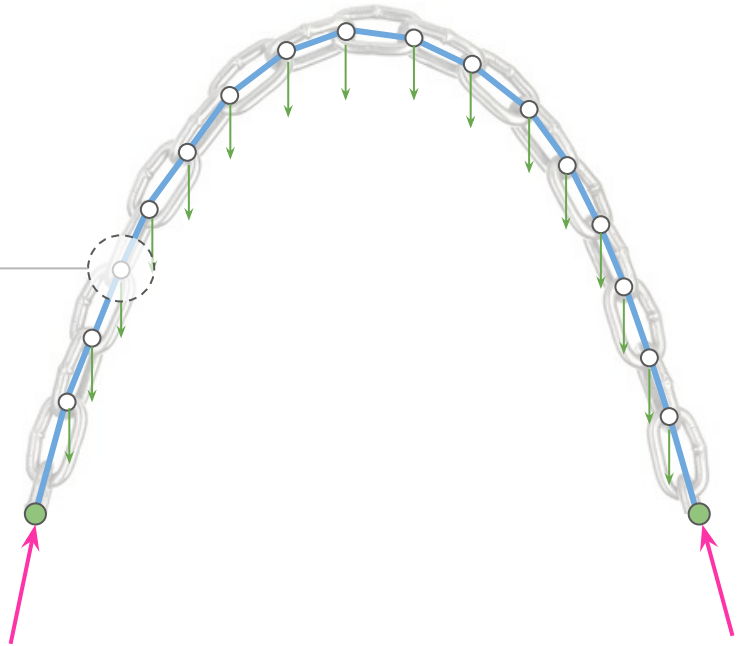
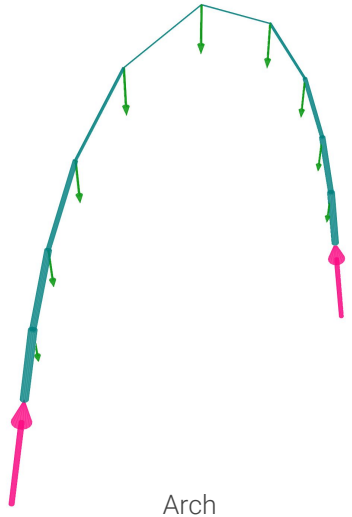


$$\sum F_i = 0$$

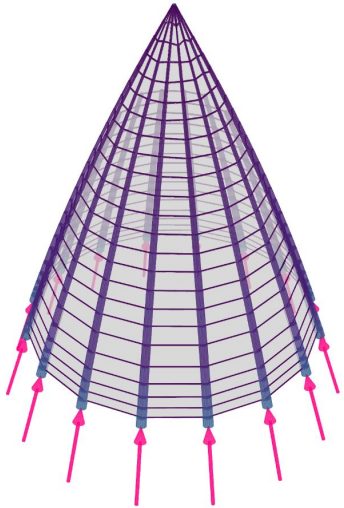


The force density method (FDM)

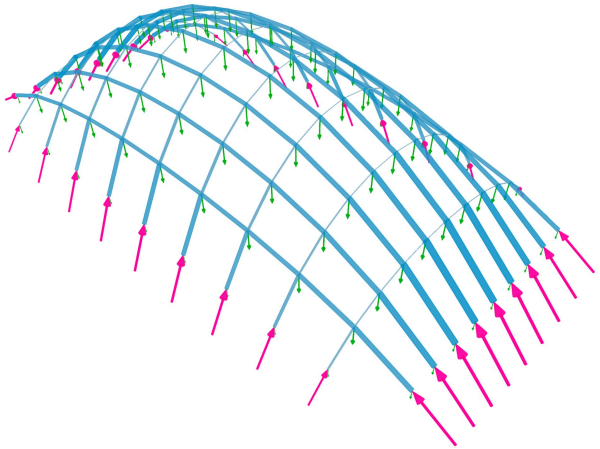
A numerical form-finding method for vaults and cable-nets



Arch



Dome

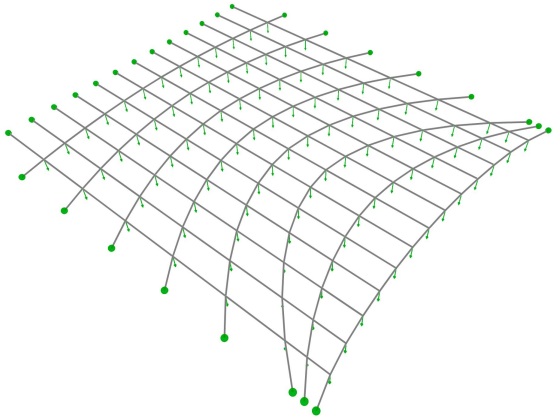


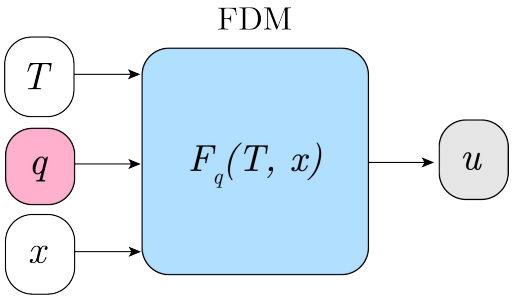
Gridshell

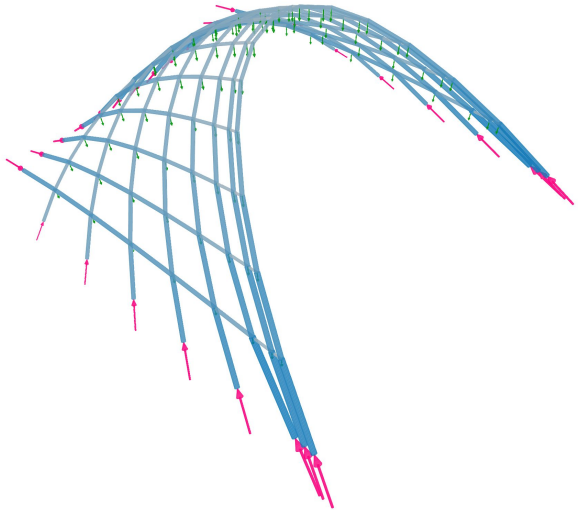
Schek (1974). The force density method for form finding and computation of general networks. DOI: 10.1016/0045-7825(74)90045-0

The force density method (FDM)

How does it work? Forward



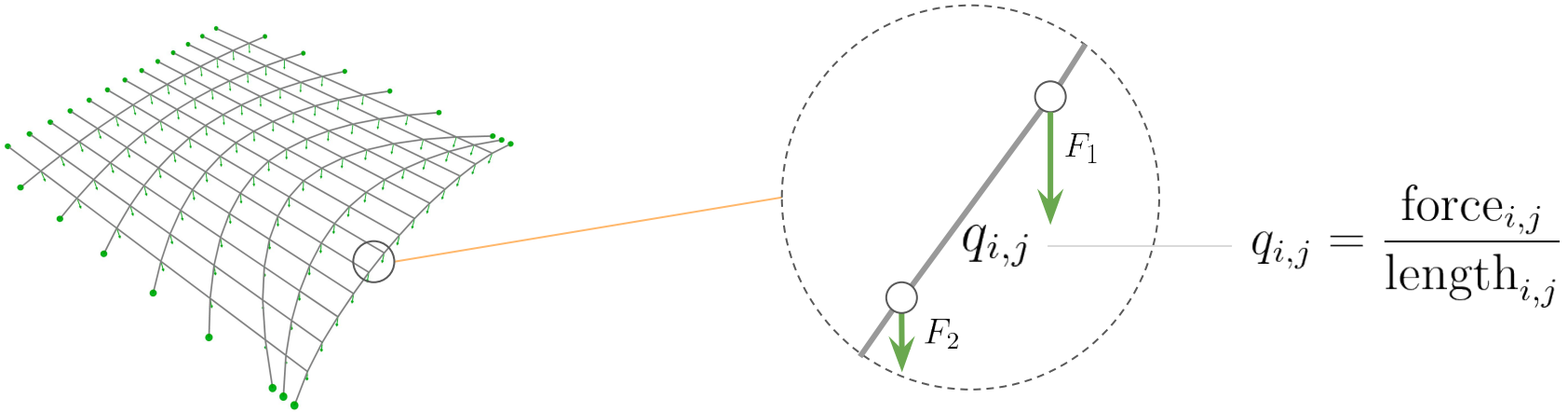




Schek (1974). The force density method for form finding and computation of general networks. DOI: 10.1016/0045-7825(74)90045-0

The force density method (FDM)

How does it work? Define the force density of the bars



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The force density method (FDM)

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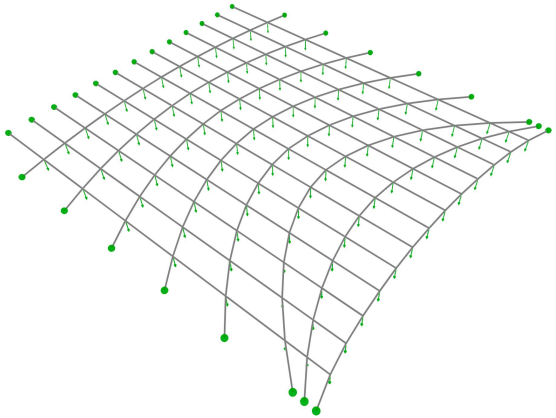

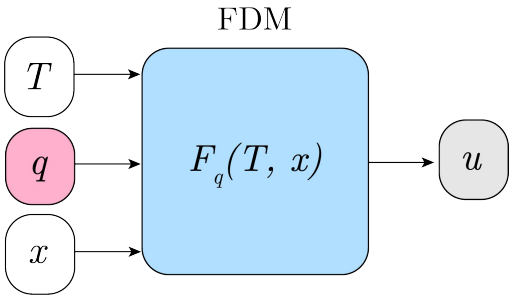

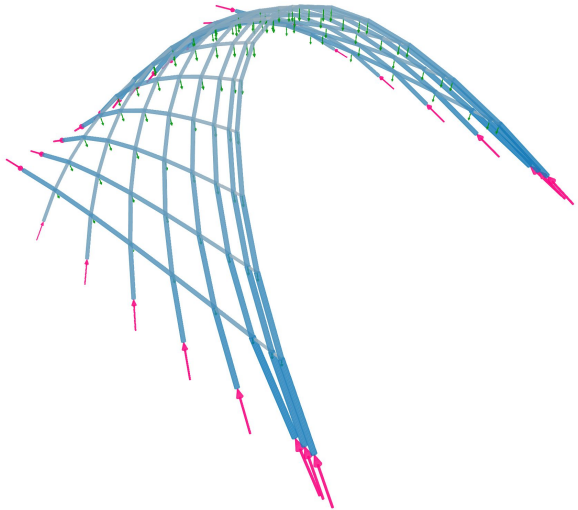

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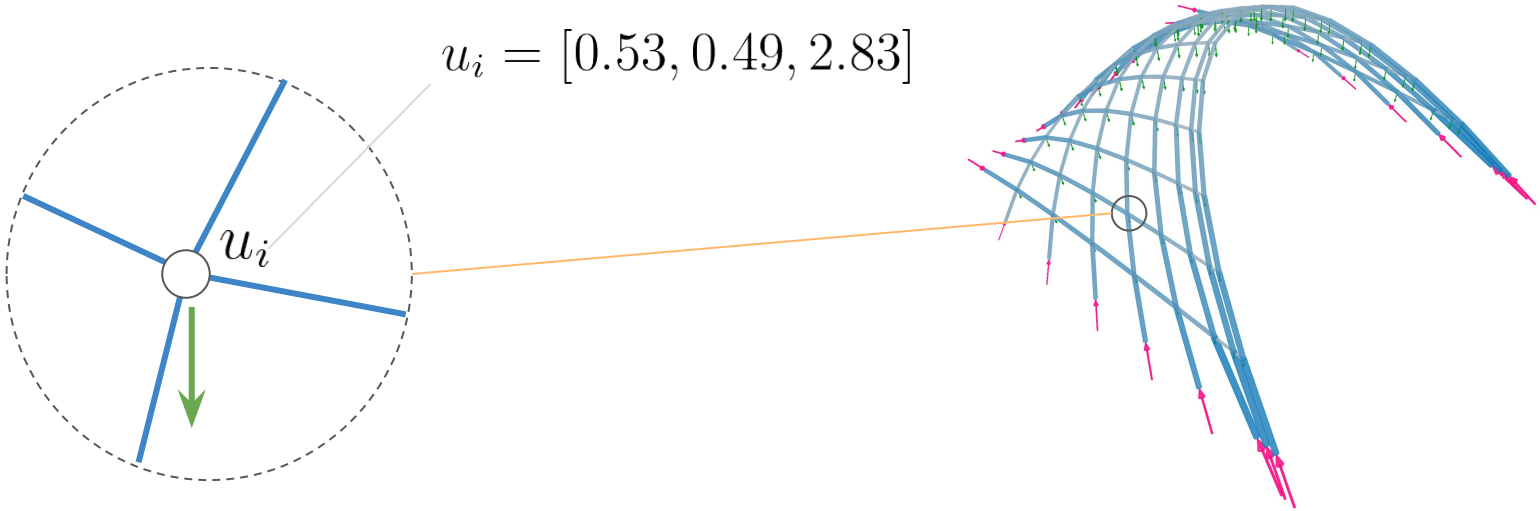
The force density method (FDM)

How does it work? Get the XYZ coordinates of the nodes



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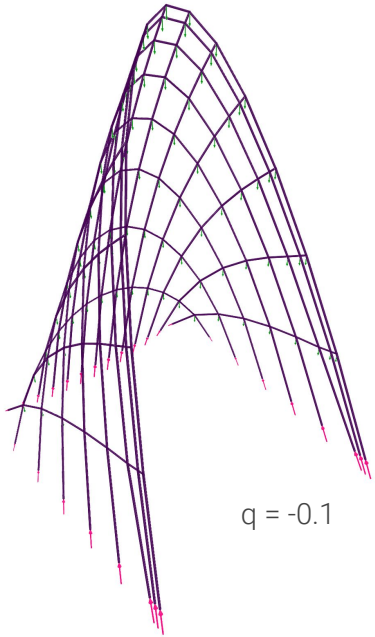
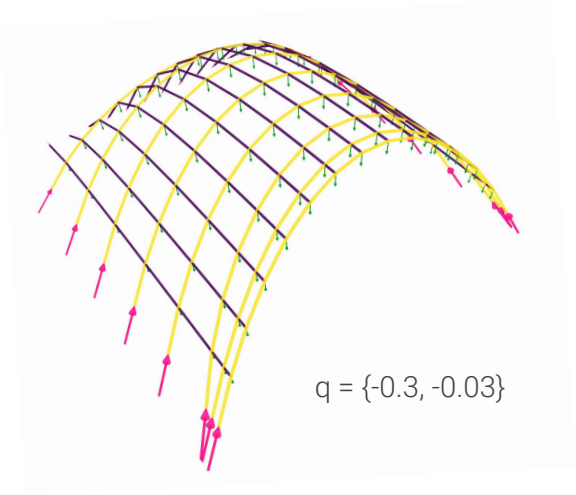
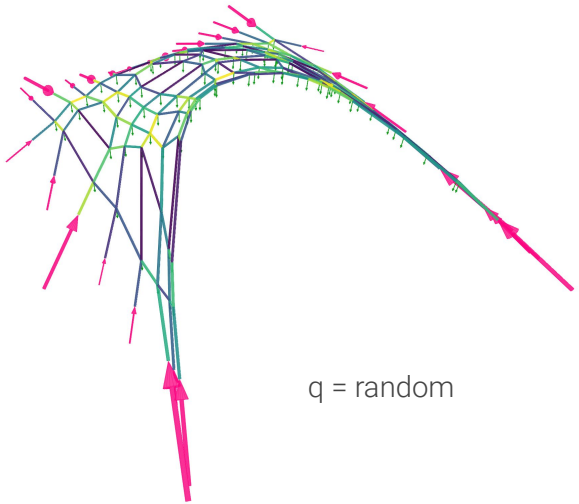
The force density method (FDM)

A plethora of shapes in static equilibrium for different force densities







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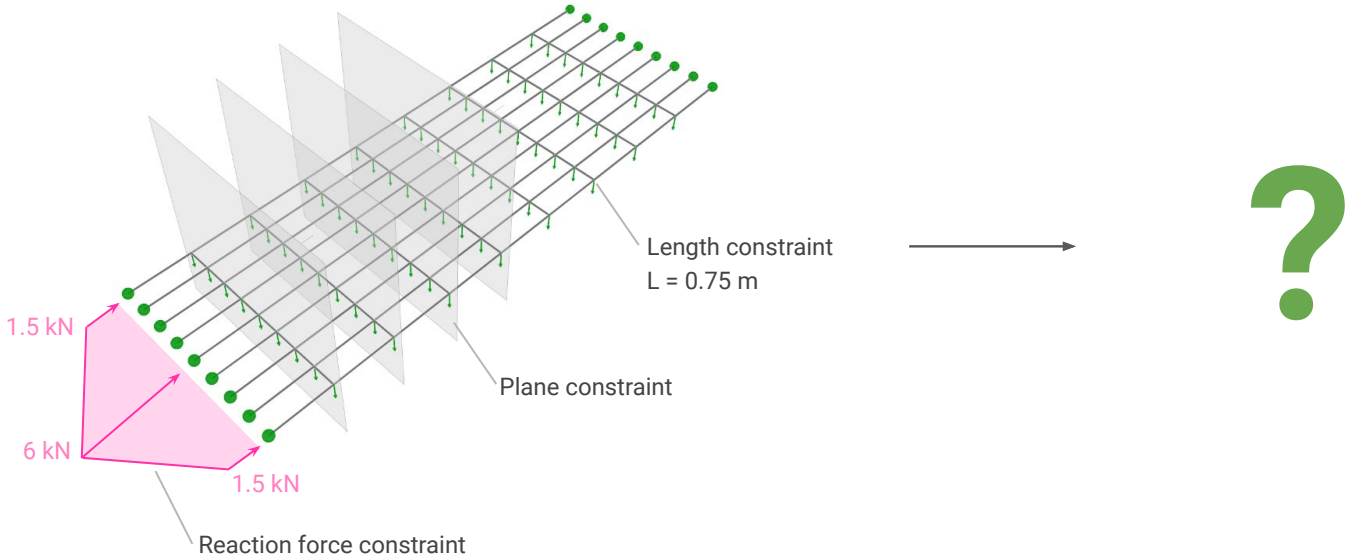
The challenges of form-finding

Viable shapes in static equilibrium meet additional constraints: fabrication, structural, aesthetic, and environmental.

No two structures are (or want to be) alike.

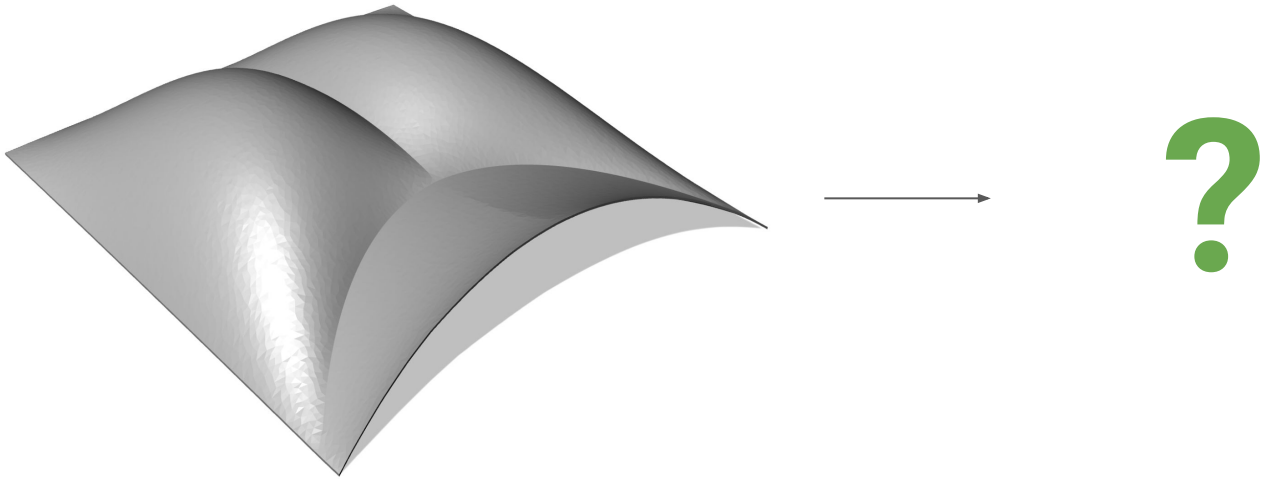
The challenges of form-finding

Shapes in equilibrium conditioned on fabrication constraints



The challenges of form-finding

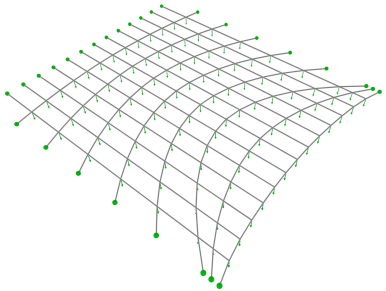
Capturing architectural design intent



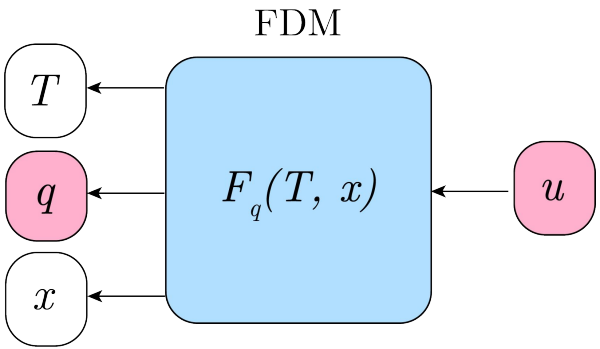
Panozzo, et al. (2011). Designing Unreinforced Masonry Models. DOI: 10.1145/2461912.2461958

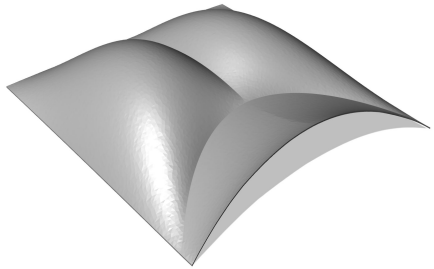
Constrained form-finding is the challenge

We actually want to solve an inverse problem



"What is the set of force densities q that are best conducive to it?"





"A shape in static equilibrium which approximates this other shape"

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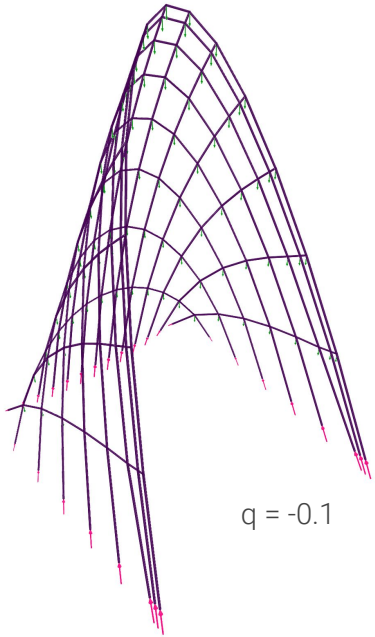
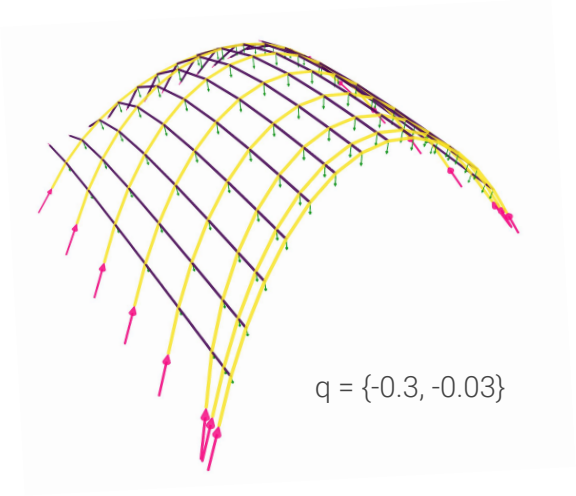
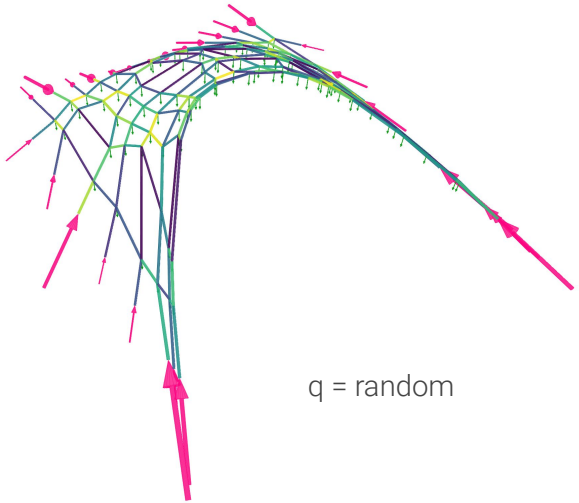
Constrained form-finding by hand?

Constrained form-finding by hand is laborious and error prone







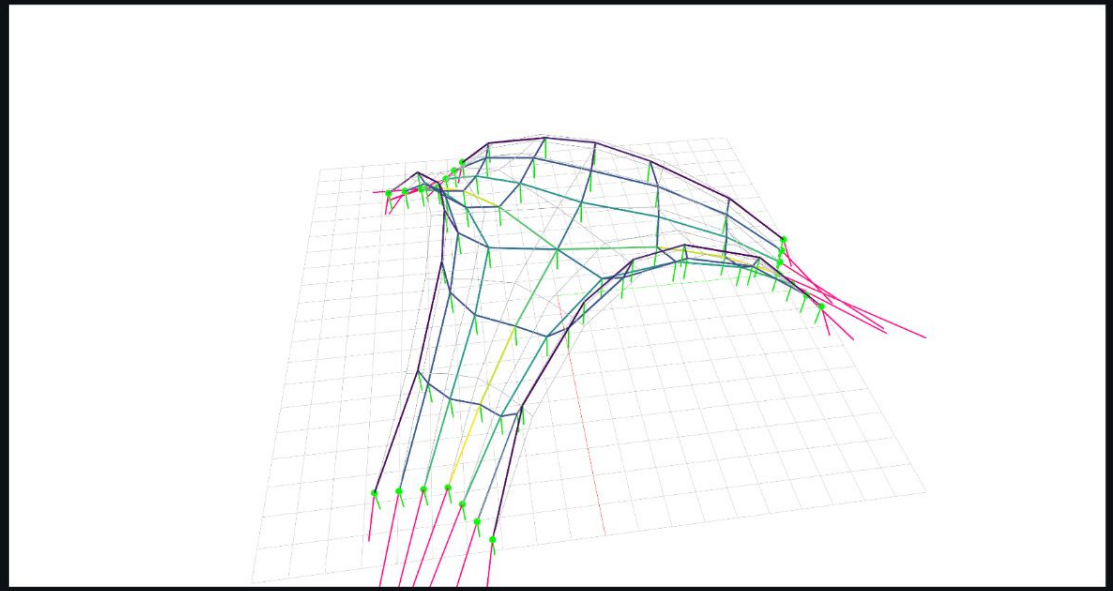
Schek (1974). The force density method for form finding and computation of general networks. DOI: 10.1016/0045-7825(74)90045-0

README.md

JAX FDM

A differentiable, hardware-accelerated framework for constrained form-finding in structural design.

> Crafted with care in the Form-Finding Lab at Princeton University ❤️🇺🇸

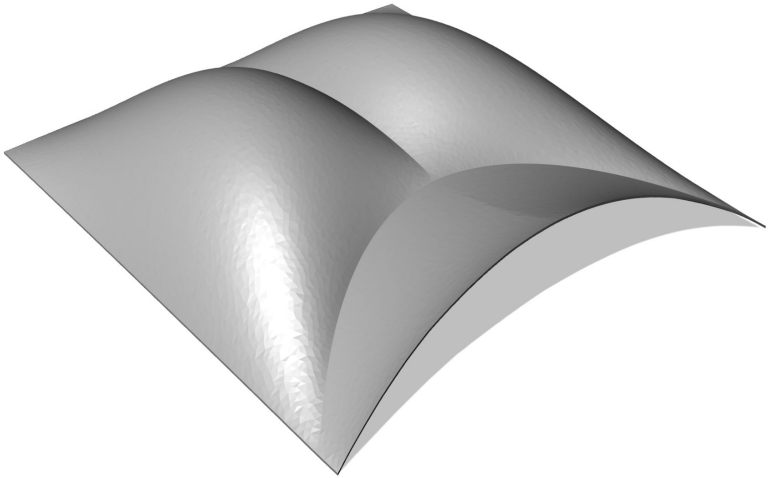


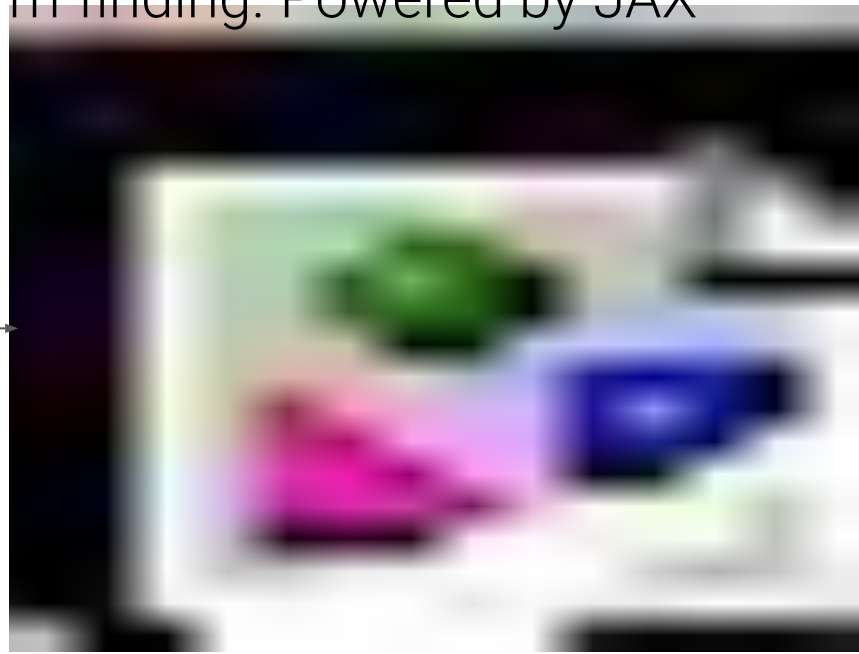
JAX FDM enables the solution of inverse form-finding problems for discrete force networks using the force density method (FDM) and gradient-based optimization. It streamlines the integration of form-finding simulations into deep learning models for machine learning research.

Key features

JAX FDM A solution

A differentiable tool for constrained form-finding. Powered by JAX





Pastrana, et al. (2022). JAX FDM: A differentiable framework for constrained form-finding. URL: https://github.com/arpastrana/jax_fdm

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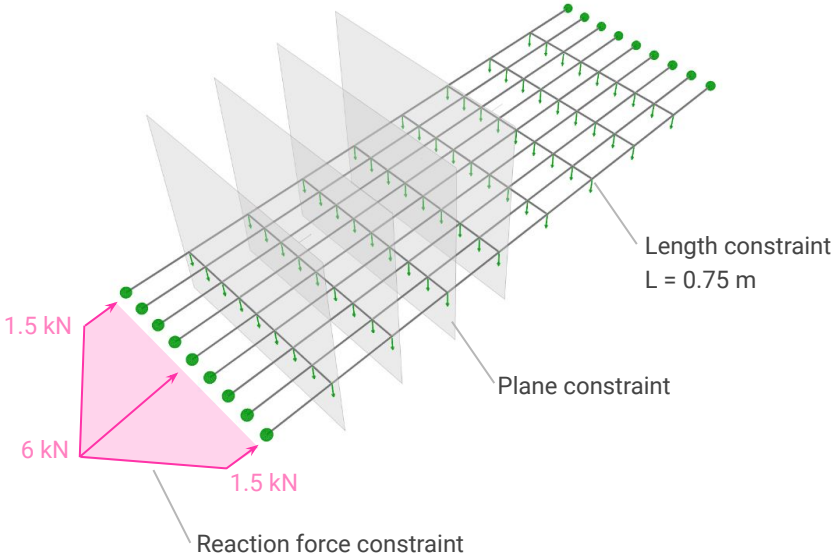


Pastrana, et al. (2022). JAX FDM: A differentiable framework for constrained form-finding. URL: https://github.com/arpastrana/jax_fdm

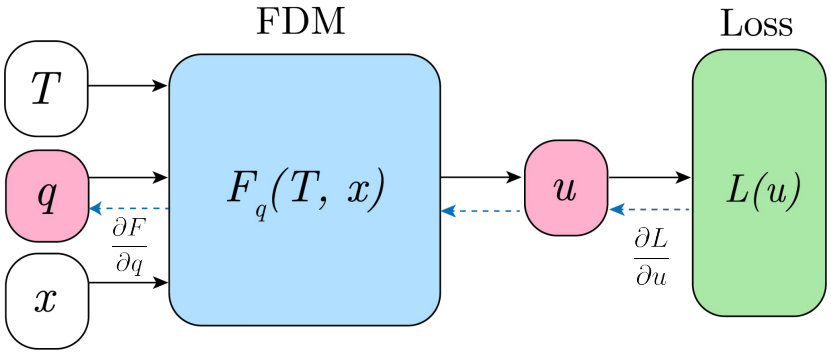
JAX FDM Tackling constrained form-finding

We apply backpropagation for the inverse design of 3D structures

$$q^t = q^{t-1} - \lambda \nabla_q L(u)$$

The new force densities

The gradient



$$L(u) = ||u - \bar{u}||^2$$

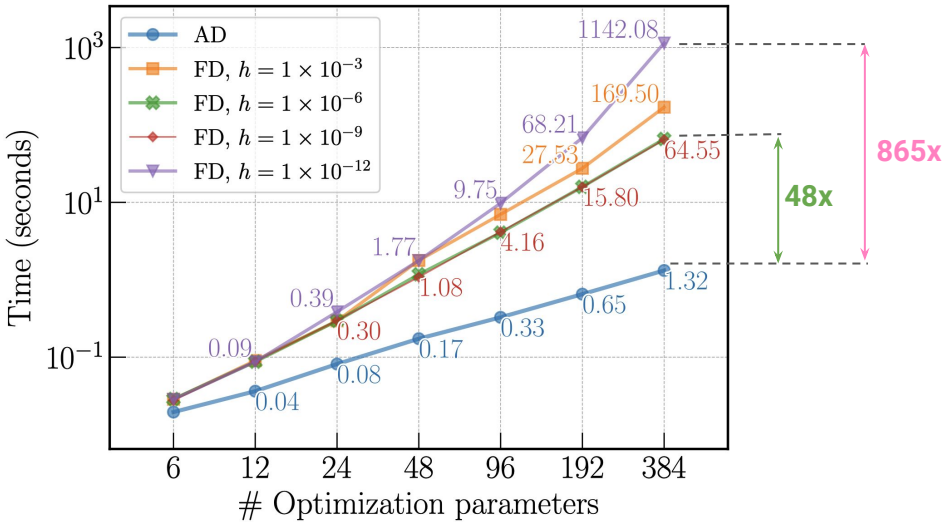
What we want

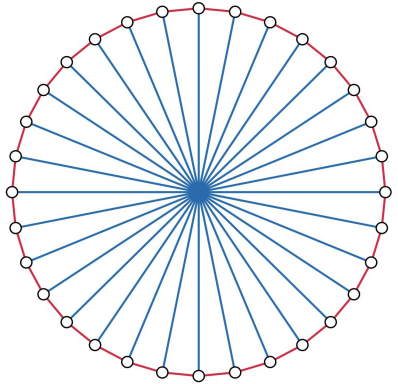
The equilibrium state

Pastrana, et al. (2022). JAX FDM: A differentiable framework for constrained form-finding. URL: https://github.com/arpastrana/jax_fdm

Constrained form-finding Backpropagation

Inverse design with backpropagation is faster and more stable





48 parameters

Pastrana, et al. (2022). Constrained form-finding of structures using automatic differentiation. Journal of computer-aided design.

```python
from jax import grad
import jax.numpy as jnp
from jax_fdm.datastructures import FDNetwork
from jax_fdm.equilibrium import EquilibriumModel

# create the FDM model
pattern, supports, loads = FDNetwork.from_json('arch.json')
model = EquilibriumModel(pattern, supports, loads)

# define the loss function
def loss(q, target_length=1.5):
   eq_state = model(q)
   return jnp.mean((eq_state.edge_lengths - target_length) ** 2)

# vanilla gradient descent
q = jnp.ones(10) * -1.0  # initial guess
lr = 0.1  # step size
for i in range(1000):
   loss_value = loss(q)
   q = q - lr * grad(loss)(q)
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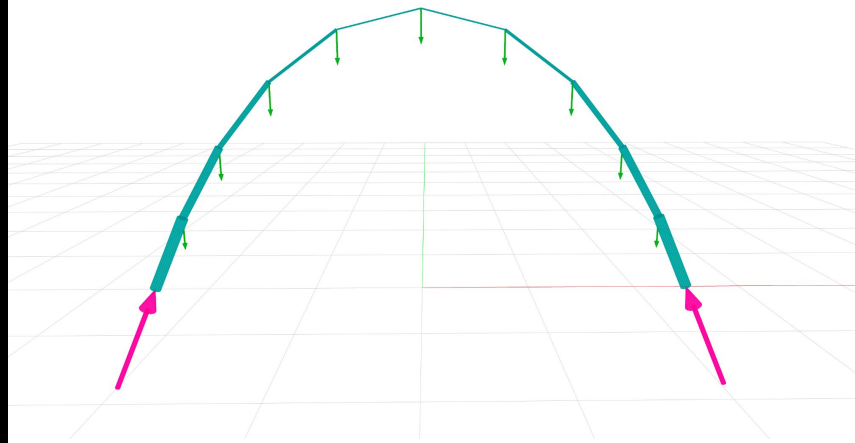
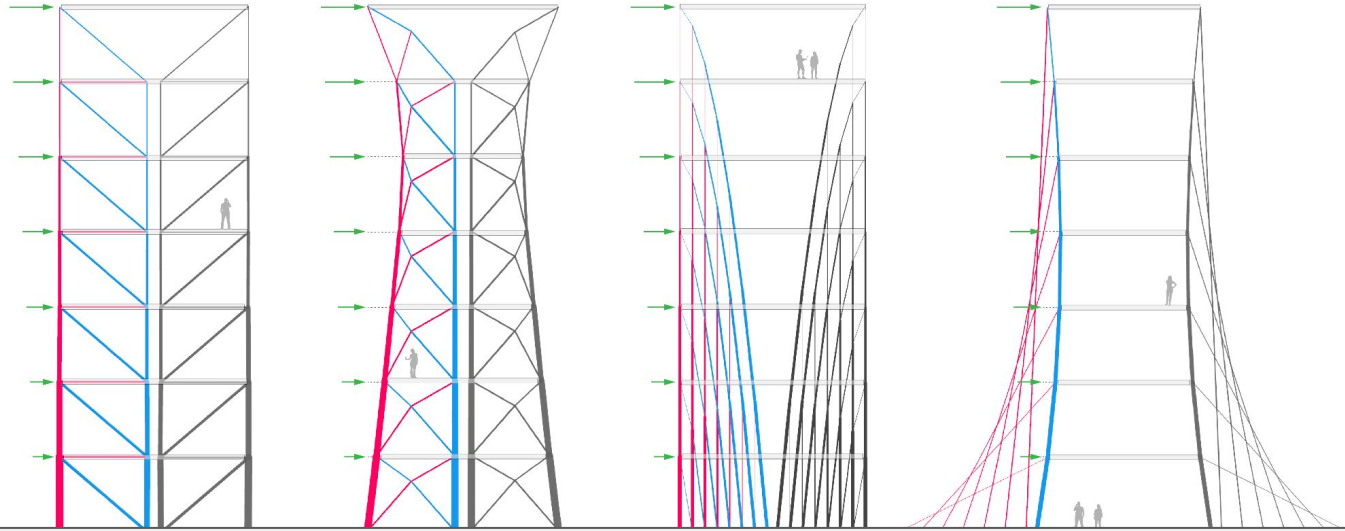


Constrained form-finding of tall buildings

Leveraging static equilibrium to design new vertical structures



Pastrana, et al. (2023). Form-finding tall buildings. Advances in Architectural Geometry. In preparation.

Constrained form-finding to reduce waste

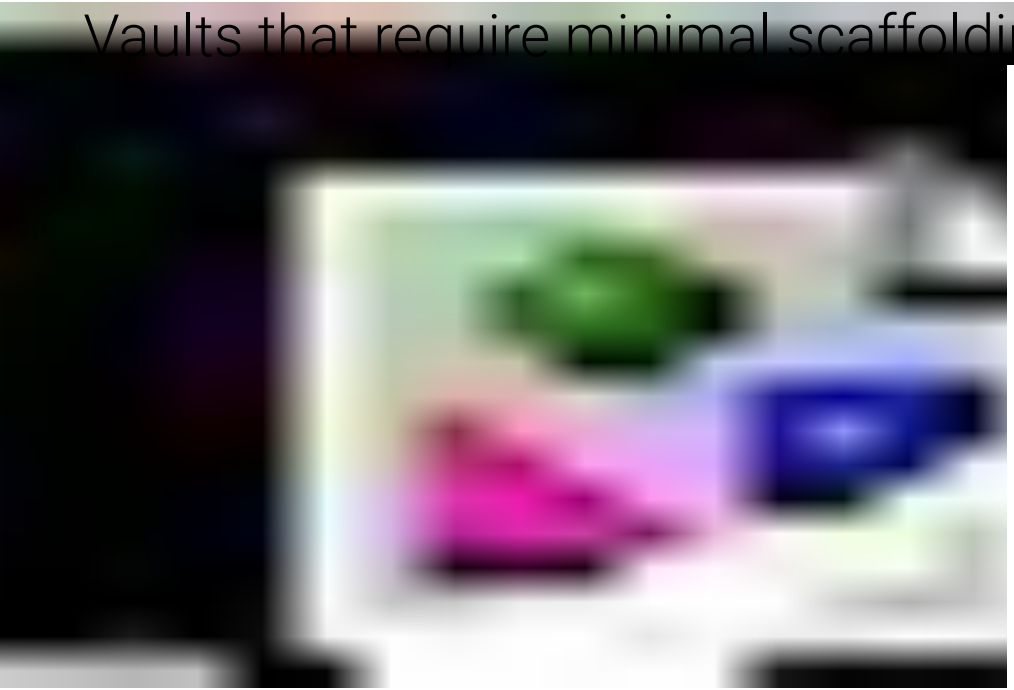
Vaults that require minimal scaffolding during construction

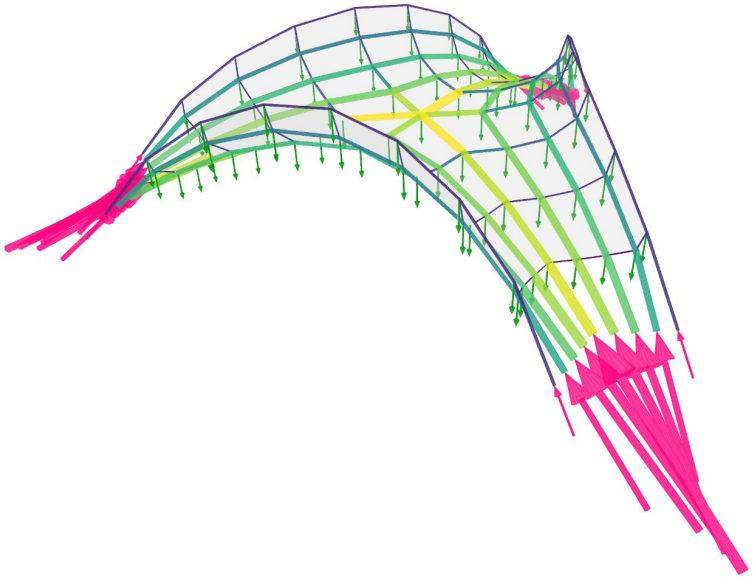
Mexican architect

Mexican builder

Constrained form-finding to reduce waste

Vaults that require minimal scaffolding during construction





Oval, et al. (2023). 4D funicular form-finding for centring-free construction. Advances in Architectural Geometry. In preparation.

JAX FDM What's next?

Combining machine learning and differentiable form-finding

Learning to solve the inverse problem

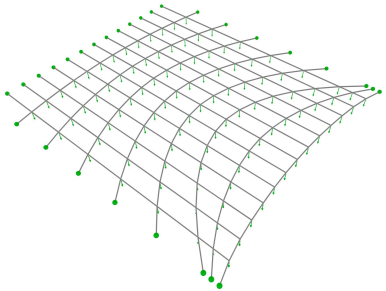
JAX FDM as a layer in a neural network. 10x speed-up?

Learning continuous representations of graphs

Can we optimize patterns and force densities jointly?

Constrained form-finding is the challenge

We actually want to solve an inverse problem



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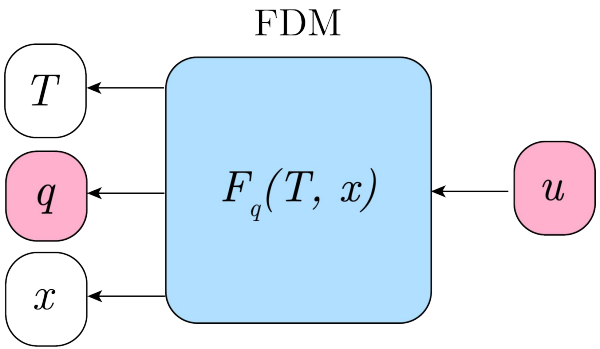

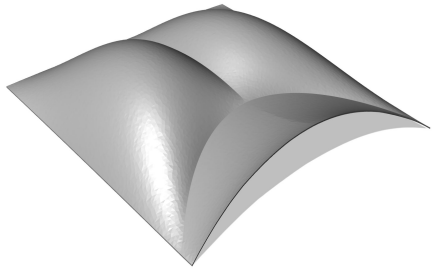

"A shape in static equilibrium which approximates this other shape"

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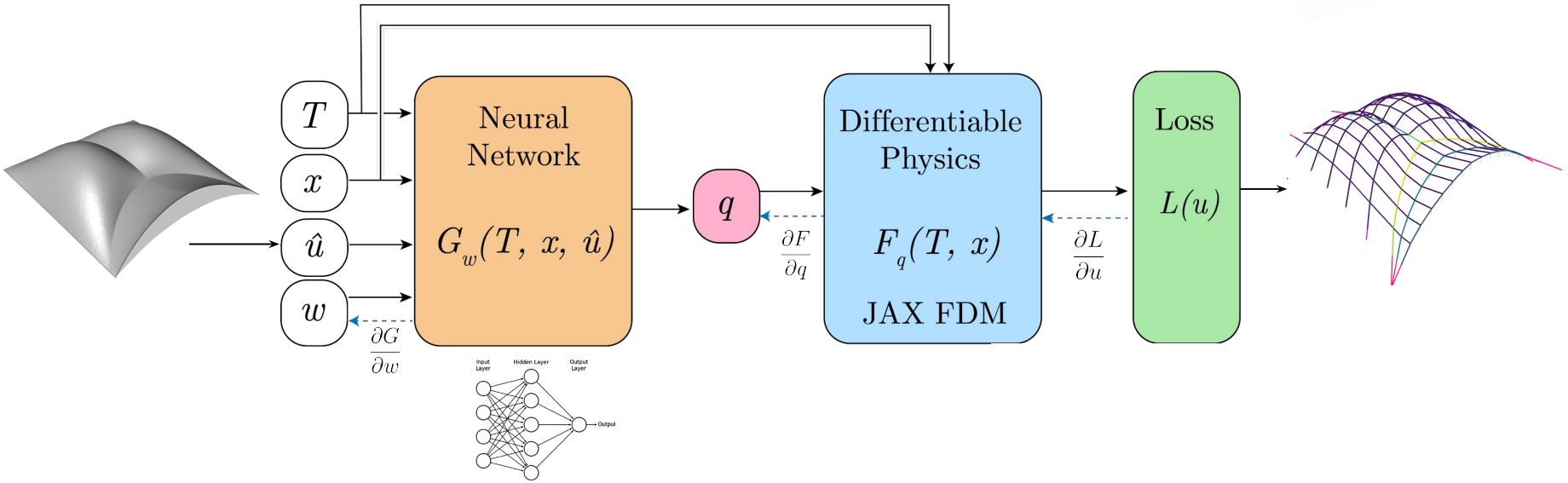
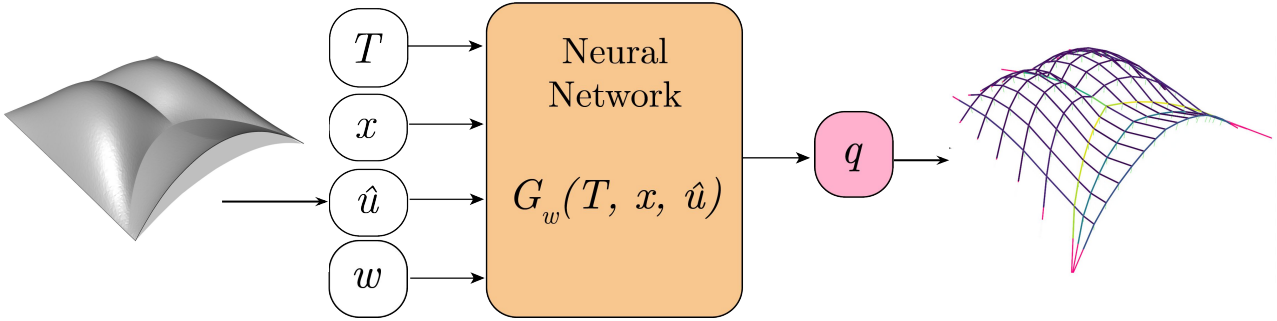
Next steps Learning to solve the inverse problem

Differentiable form-finding as a layer in a neural network



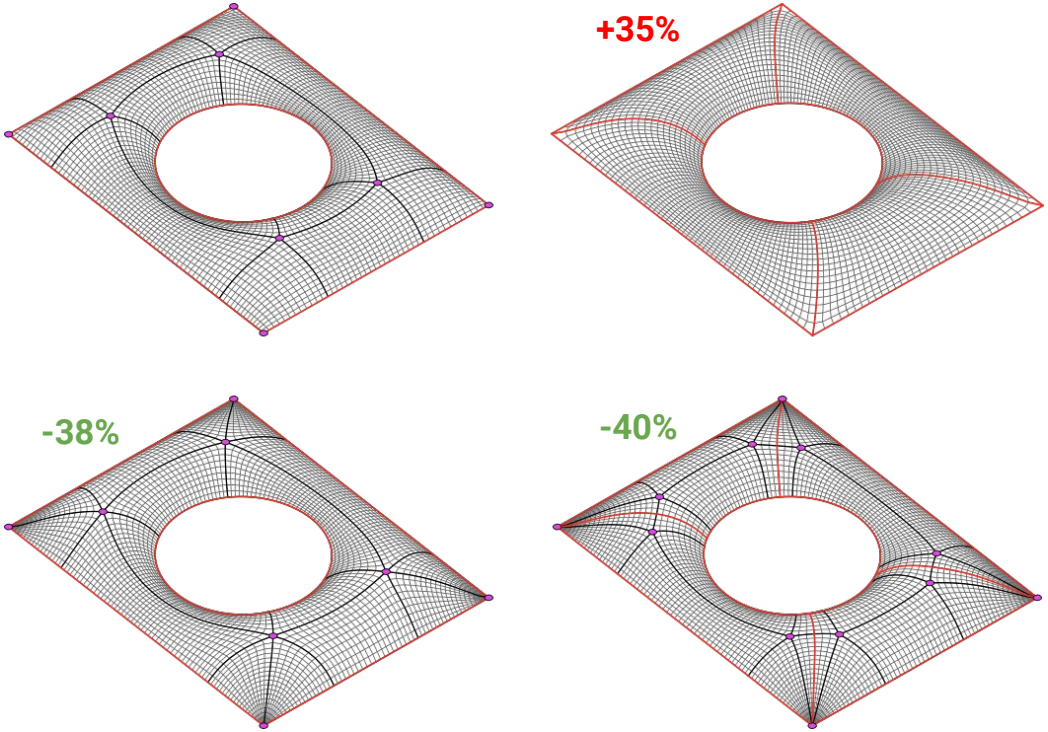
Next steps Learning to solve the inverse problem

Differentiable form-finding as a layer in a neural network



10x speed-up?

Next steps Generative model of patterns

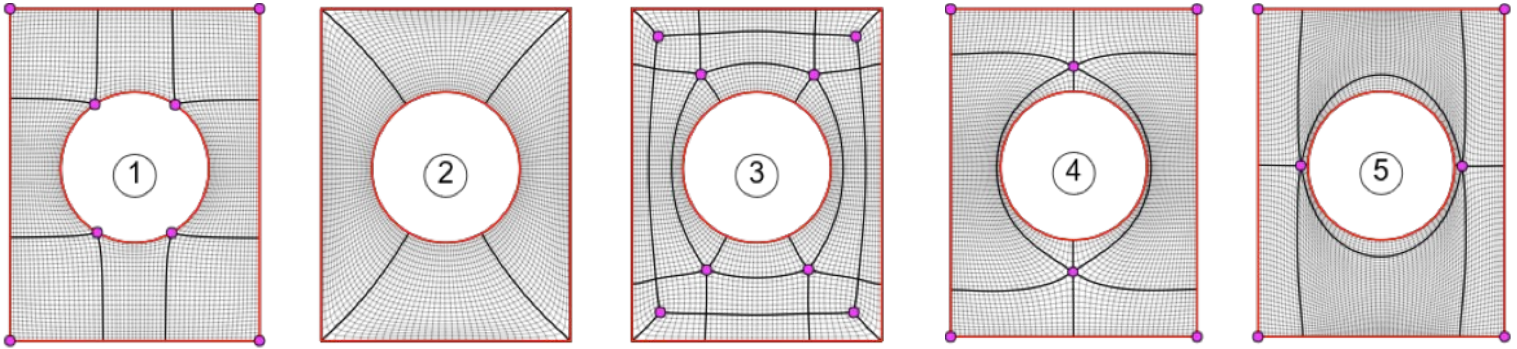


Oval (2019). Topology finding of patterns for structural design. Ph.D. thesis

Next steps Generative model of patterns

Learning continuous representations of graphs

Cast as a **NLP** task?



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Next steps Generative mo el of patterns

Learning continuous repr

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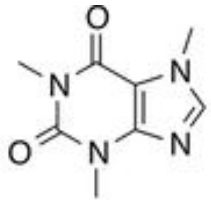

Caffeine

CN1c2ncn(C)c2C(=O)N(C)C1=O

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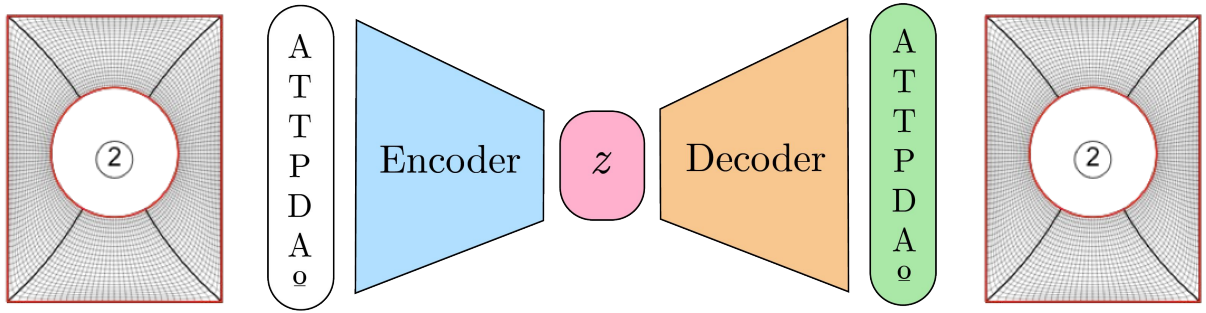
Next steps Generative model of patterns

Learning continuous representations of graphs

Cast as a **NLP** task?



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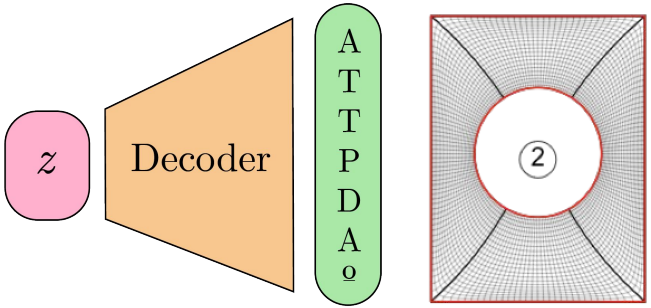
Learning continuous representations of graphs

Cast as a **NLP** task?



Oval (2019). Topology finding of patterns for structural design. Ph.D. thesis

JAX FDM

A differentiable framework for constrained form-finding

Rafael Pastrana, Sigrid Adriaenssens

https://github.com/arpastrana/jax_fdm

Princeton University
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FORM
FINDING
Lab