I. (20) Beside each statement place a T if true and an F if false.

**F** 1. Increasing the distance between two point charges by a factor of 2, decreases the electric force between them by a factor of 16.

**F** 2. A point charge $+q$ is at the origin, and two spherical Gaussian surfaces are centered at the origin. The Gaussian surface with the larger radius has a greater electric flux through it.

**T** 3. Electric field lines originate or begin on positive charges.

**T** 4. Gauss's Law can be derived from Coulomb's Law.

**F** 5. Work is done when a charge moves along an equipotential contour.

**T** 6. A charged particle is moved between two points in a region of electric field. The work done is the same for every path taken between the two points.

**F** 7. If the plates of an isolated parallel plate capacitor are moved further apart, the stored energy in the capacitor decreases.

**T** 8. When a dielectric is inserted between the plates of a parallel plate capacitor, the capacitance increases.

**T** 9. The electric flux is a scalar quantity.

**F** 10. An ideal voltmeter would have zero resistance.

**F** 11. The volt is equivalent to a J/N.

**F** 12. An electric dipole is a pair of equal positive charges separated by some distance.

**T** 13. Capacitors connected in series have the same charge.

**T** 14. Semi-conductors generally have a negative temperature coefficient of resistivity.

**F** 15. For resistors connected in parallel, more current will flow through the largest resistor.

**F** 16. Kirchhoff's current law or rule at a junction is a consequence of conservation of energy.

**T** 17. The resistance of a resistor is equal to the slope of a graph of voltage plotted along the vertical (y) axis and current along the horizontal (x) axis.

**T** 18. The Wheatstone Bridge is a circuit that can be used to measure an unknown resistance.

**T** 19. A capacitor behaves as an open circuit after very long times in an RC circuit.

**F** 20. If the current in a resistor doubles, the power level in the resistor also doubles.

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100 pts
II. (20) A wire in the shape of a circular arc of radius $R$ and 90 degrees total angle has a uniform linear charge density $\lambda$ (in C/m). Find the electric field $\mathbf{E}$ at point P $(0, R/2)$ shown in the figure below. Reduce the calculation to fully defined integrals, but do not integrate. Give all reasoning and show all work.

\[ \mathbf{r} = \frac{R}{2} \hat{\mathbf{y}} - \left( + R \cos \theta \hat{\mathbf{i}} + R \sin \theta \hat{\mathbf{j}} \right) \]
\[ = \left( - R \cos \theta \right) \hat{\mathbf{i}} + \left( \frac{R}{2} - R \sin \theta \right) \hat{\mathbf{j}} \]
\[ \mathbf{r} = \sqrt{R^2 \cos^2 \theta + \left( \frac{R}{2} - R \sin \theta \right)^2} \]
\[ = \sqrt{R^2 + \frac{R^2}{4} - R^2 \sin^2 \theta} = R \sqrt{\frac{5}{4} - \sin^2 \theta} \]
\[ \hat{\mathbf{r}} = \frac{- \cos \theta \hat{\mathbf{i}} + \left( \frac{1}{2} - \sin \theta \right) \hat{\mathbf{j}}}{\sqrt{\frac{5}{4} - \sin^2 \theta}} \]

\[ \mathbf{E}_P = \frac{1}{4\pi \varepsilon_0} \int \frac{\lambda R \, d\theta}{r^2} \hat{\mathbf{r}} = \frac{1}{4\pi \varepsilon_0} \lambda R \int_0^{\pi/2} \frac{- \cos \theta \hat{\mathbf{i}} + \left( \frac{1}{2} - \sin \theta \right) \hat{\mathbf{j}}}{\sqrt{\frac{5}{4} - \sin^2 \theta}} R^2 \left( \frac{5}{4} - \sin^2 \theta \right) \, d\theta \]
\[ \mathbf{E}_P = \frac{\lambda}{4\pi \varepsilon_0 R} \int_0^{\pi/2} \frac{- \cos \theta \hat{\mathbf{i}} + \left( \frac{1}{2} - \sin \theta \right) \hat{\mathbf{j}}}{\left( \frac{5}{4} - \sin^2 \theta \right)^{3/2}} \, d\theta \]

III. (20) Short Answer. Briefly calculate or describe in words the fundamental physics principles and reasoning which govern the following situations. Include any information you think is important. Write legibly and use complete sentences.

1. (5) Explain the relationship between the equipotential contours around a point and the electric field $\mathbf{E}$ at that point.

1. $\mathbf{E}$ is perpendicular to the equipotential contours.
2. $\mathbf{E}$ points from high equipotential to low equipotential.
3. $\mathbf{E}$ is larger where the equipotential contours are closer together.
4. $E_x = -\frac{dV}{dx}$ or $\mathbf{E} = -\nabla V$ gradient (or $V_A - V_B = \int_B^A \mathbf{E} \cdot d\mathbf{r}$)
2. A. (3) Describe the properties of the device called a thermistor and discuss what it can be used to measure.

A thermistor is a (semi-conducting) device whose resistance varies with temperature (ΔR ↔ ΔT). When calibrated, its resistance can be used to measure temperature. (The resistance variation (ΔR) can be converted to a variation of current or voltage with a Wheatstone Bridge Circuit.)

B. (3) How does an electro scope work and what does it measure?

An electro scope can be used to measure charge. When the electro scope is charged, two metal parts acquire like charges (both + or both -). Since one of the metal parts (a rod or needle) can swing, the like charges create a repulsion which swings the rod or needle out. The amount of deflection can be related to charge.

3. (4) It is very common to use extremely high voltages (up to 750 kV), rather than low voltages (such as 120 V), to transmit electrical power over very long conducting transmission lines. Provide a reason for this.

Power Transmitted = IV

Power Loss = I²R

To reduce power loss during transmission, reduce I, and therefore, increase V.

4. (5) In Lab II the force between plates was given by \( F = \frac{\varepsilon_0 A V^2}{2 D^2} \). Suppose the error in the measurement of V is ΔV. Find ΔF induced by this error ΔV.

See page 7 in lab manual.

\[ \Delta F_V = \left| F(V+\Delta V) - F(V) \right| = \left| \frac{\varepsilon_0 A (V+\Delta V)^2}{2 D^2} - \frac{\varepsilon_0 A V^2}{2 D^2} \right| \]
IV. (20) The inner radius of a spherical insulating shell is \( R_1 \) and the outer radius is \( R_2 \). The shell has a constant volume charge density \( \rho = \rho_0 \) (in \( \text{C/m}^3 \)) for \( R_1 \leq r \leq R_2 \), where \( r \) is the distance from the center of the shell.

\[ \text{Cross-section of spherical shell} \]

\[ \text{Gaussian Surface (Sphere) - area is} \quad 4\pi r^2 \]

1. (10) Starting from Gauss’ Law find the magnitude of the electric field \( \vec{E} \) for \( R_1 \leq r \leq R_2 \) and also state what the direction of \( \vec{E} \) is. Give all reasoning and show all work.

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

\[ \vec{E} \] is the same at all \( \vec{dA} \)

\[ \oint \vec{E} \cdot \cos \theta d\vec{A} = \]

\[ E \oint d\vec{A} \]

\[ E = \frac{Q_{\text{enc}}}{4\pi \varepsilon_0 r^2} \]

\[ Q_{\text{enc}} = \int_{R_1}^{R_2} \rho_0 4\pi r^2 dr \]

\[ Q_{\text{enc}} = \rho_0 \left( \frac{4}{3} \pi R_3^3 - \frac{4}{3} \pi R_1^3 \right) \]

(you can get \( Q_{\text{enc}} \) without integrating if you know the volume of a sphere)

\[ E_{\text{radial}} = \left( \frac{\rho_0 r}{3 \varepsilon_0} - \frac{\rho_0 R_1^3}{3 \varepsilon_0 r^2} \right) \hat{r} \]

Put it together:

\[ E = \left( \frac{\rho_0 r}{3 \varepsilon_0} - \frac{\rho_0 R_1^3}{3 \varepsilon_0 r^2} \right) \hat{r} \quad \text{radially outward} \]

2. (10) If the electric potential at the center of the shell is taken as zero, find the electric potential at the outer radius, \( r = R_2 \), of the shell. Give all reasoning and show all work.

\[ V_A - V_B = \int_{B}^{A} \vec{E} \cdot d\vec{s} \]

You first must think about \( \vec{E} \) for \( 0 \leq r \leq R_1 \). Since there is no charge inside the cavity of the shell, \( Q_{\text{enc}} = 0 \) for Gaussian surface inside \( (0 \leq r \leq R_1) \). Therefore \( \vec{E} = 0 \) in the cavity.

Point A \( r = R_2 \), point B \( r = 0 \)

\[ \vec{d\vec{s}} = \hat{r} dr \]

\[ VR_2 - V_0 = \int_{0}^{R_1} 0 \cdot \hat{r} dr - \int_{R_1}^{R_2} \left( \frac{\rho_0 r}{3 \varepsilon_0} - \frac{\rho_0 R_1^3}{3 \varepsilon_0 r^2} \right) \hat{r} \cdot \hat{r} dr \]

\[ VR_2 = \frac{-\rho_0}{3 \varepsilon_0} \left( \frac{R_2^2}{2} - \frac{R_1^2}{2} \right) \]

\[ \frac{-\rho_0 R_1^3}{3 \varepsilon_0} \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \]

\[ V_{R_2} = \]
V. (20) Consider the circuit shown below. Explain all reasoning, state all assumptions, and show all work on each part.

\[ \begin{align*}
30.0 \text{V} & \quad \text{R}_1 \quad 10.0 \Omega \\
\varepsilon & \quad \text{R}_2 \quad 10.0 \Omega \\
\text{S} & \quad \text{R}_3 \quad 10.0 \Omega \\
& \quad 0.200 \mu \text{F} \\
\end{align*} \]

(DATA: \( \varepsilon = 30.0 \text{ V}, \text{R}_1 = 10.0 \Omega, \text{R}_2 = 10.0 \Omega, \text{R}_3 = 10.0 \Omega, \text{C} = 0.200 \mu \text{F} \), the capacitor is initially uncharged, and the switch \( \text{S} \) is closed at \( t = 0 \).)

1. (4) Find the numerical values of the current through \( \text{R}_2 \) and the voltage across it at \( t = 0 \), when the switch is closed.

At \( t = 0 \), \( \text{Q}_c = 0 \Rightarrow \text{V}_c = 0 \) (\( \text{C} \) acts as shorted)

\[ \begin{align*}
\text{I}_{\text{R}_2}(0) & = 1.00 \text{ A} \\
\text{V}_{\text{R}_2}(0) & = 10.0 \text{ V} \\
\end{align*} \]

2. (4) Find the numerical values of the current through \( \text{R}_2 \) and the voltage across it after a very long time.

\( \text{No current through } \text{R}_3 \text{ at } t \rightarrow \infty. \)

\[ \begin{align*}
\text{I}_{\text{R}_2}(\infty) & = 1.50 \text{ A} \\
\text{V}_{\text{R}_2}(\infty) & = 15.0 \text{ V} \\
\end{align*} \]

3. (8) Write below enough equations (including Kirchhoff's Rules) which could be used to completely solve this circuit (find all voltages and currents as a function of time). Use symbols in this part (\( \text{R}_1 \) instead of \( 10.0 \Omega \)), and you DO NOT need to solve the equations.

\[ \begin{align*}
\varepsilon - \text{I}_1 \text{R}_1 - \text{I}_2 \text{R}_2 & = 0 \\
- \text{I}_3 \text{R}_3 - \frac{\text{Q}}{\text{C}} + \text{I}_2 \text{R}_2 & = 0 \\
\text{I}_1 & = \text{I}_2 + \text{I}_3 \\
\text{I}_3 & = \frac{d\text{Q}}{dt} \\
\end{align*} \]
4. (4) On the graph below show qualitatively what the voltage across the capacitor would look like as a function of time (t = 0 to t = a very long time). Also, indicate on the y-axis the numerical value of the capacitor voltage after a very long time.