AP Fluids Problems

1. The atmosphere has a thickness of around 10 miles. With that knowledge and assuming a constant density, calculate how many atms we feel here in Castle Rock (Elevation = 6224'). **(0.882 atm)**

$6224\,ft\left(\frac{1\,mi}{5280\,ft}\right) = 1.18\,mi$ above sea level, so that means there is 8.82 mi of air above us.

$\frac{1\,atm}{10\,mi} = \frac{P_{Castle Rock}}{8.82} = \boxed{0.882\,atm}$

2. The human body, for a teenager, has a surface area of around 1.5 m^2. Calculate how many pounds of force the atmosphere is trying to squeeze you with right now. (1 lb = 4.45 N) **(30,121 lbs)**

$0.882\,atm\left(\frac{101300\,Pa}{1\,atm}\right) = 89{,}359\,Pa = 89{,}359\,N/m^2$

$F = P \cdot A = 89359 \cdot 1.5 = 134{,}039\,N \rightarrow \boxed{30{,}121\,lbs}$

3. A cylinder with radius 15 cm and height of 40 cm has a mass of 14.14 kg. If placed in water (ρ = 1 g/cm^3), how much of the cylinder is floating in the water? **(20 cm)**

$\rho = m/v = \frac{14140\,g}{\pi(15)^2(40)} = 0.5\,g/cm^3$ which is less than ρ_{water} ∴ floats

if Floating $F_B = F_g$

$m_w g = m_{cyl} g$

$m_w = m_{cyl}$

$\rho_w V_w = m_{cyl}$

$V_w = m/\rho = \frac{14140}{1} = 14140\,cm^3$

$h = V/A = \frac{14140}{\pi(15)^2} = \boxed{20\,cm}$

4. Each second, 5,525 m^3 of water flows over the 670 m wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is about 2 m deep as it reaches the cliff. Calculate the speed of the water. **(4.12 m/s)**

$\frac{\Delta Vol}{\Delta t} = 5525\,m^3/s$

Area = width · depth

$= 670 \cdot 2 = 1340\,m^2$

$\frac{\Delta Vol}{\Delta t} = A \cdot v \Rightarrow v = \left(\frac{\Delta Vol}{\Delta t}\right)/A = \frac{5525}{1340} = \boxed{4.12}$

5. The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body.
   a. Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (1 mL = 1 cm^3) **(0.27 m/s)**

$\frac{\Delta V}{\Delta t} = 5\frac{L}{min}\left(\frac{1000\,mL}{1\,L}\right)\left(\frac{1\,min}{60\,sec}\right)\left(\frac{1\,cm^3}{1\,mL}\right) = 83.3\,\frac{cm^3}{s} = A \cdot v \Rightarrow v = \frac{83.3}{\pi(1\,cm)^2} = 26.5\,cm/s$

$= \boxed{0.27\,m/s}$

   b. Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0μm, calculate the number of capillaries in the blood circulatory system. **(5 billion capillaries)**

flow rate = $A \cdot v$

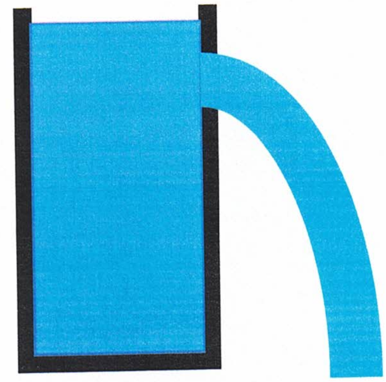
for 1 capillary $= \pi\left(\frac{8 \cdot 10^{-4}\,cm}{2}\right)^2 (.033\,\frac{cm}{s})$

$= 1.659 \cdot 10^{-8}\,\frac{cm^3}{s}$

$N \cdot (\text{flowrate}_{1\,capillary}) = $ total flow rate

$N = \frac{83.3}{1.659 \cdot 10^{-8}} = \boxed{5.02 \cdot 10^9}$

6. A nearsighted sheriff fires at a cattle thief with his trusty six-shooter. He missed and the bullet penetrates the town water tank, causing a leak.



   a. If the top of the tank is open to the atmosphere, determine the speed at which the water leaves the hole when the water level is 0.5 m above the hole. **3.13 m/s)**

$$P_1 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2$$

b/c both open to air $P_1 = P_2$

$$\cancel{\rho} g h = \tfrac{1}{2}\cancel{\rho} v^2$$

$$v = \sqrt{2gh}$$
$$= \sqrt{2(9.8)(.5)}$$
$$\boxed{= 3.13\text{m/s}}$$

   b. Where does the stream hit the ground if the hole is 3.0 m above the ground? **(2.45 m)**

x

$v_x = 3.13\text{m/s}$

$\Delta x = ? = v_x t$

$= \boxed{2.45\text{m}}$

y

$y_i = 3\text{m}$ $\quad v_i = 0$

$y_f = 0$

$a_y = -9.8$

2nd Kinematic Equation

$0 = 3 + 0t + \tfrac{1}{2}(-9.8)t^2$

$t = 0.782\text{sec}$

7. The area of each wing of a Boeing 747 is 255.48 m^2. The wings are designed so that air flows over the top of the wing at 245 m/s and underneath the wing at 222 m/s. What is the maximum weight that the wings can lift? The density of air is 1.29 kg/m3. **(3,540,000 N ≈ 800,000 lbs)**

$$P_{bottom} + \tfrac{1}{2}\rho v_{bottom}^2 = P_{top} + \tfrac{1}{2}\rho v_{top}^2$$

$$\Delta P = \tfrac{1}{2}\rho(v_{top}^2 - v_{bottom}^2)$$
$$= 6927.945 \text{ Pa}$$

this causes a lift force

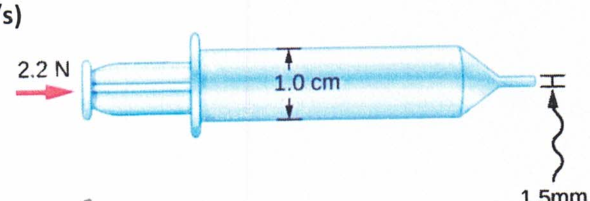
2 wings

$F = P \cdot A$

$F = (6927.945)(255.48) \times 2$

$\boxed{= 3540{,}000\text{ N} \\ = 795{,}000\text{ lbs}}$

8. A horizontally oriented squirt toy contains a 1.0-cm-diameter barrel for the water. A 2.2-N force on the plunger forces water down the barrel and into a 1.5-mm-diameter opening at the end of the squirt gun. In addition to the force pushing on the plunger, pressure from the atmosphere is also present at both ends of the gun, pushing the plunger in and also pushing the water back in to the narrow opening at the other end. Assuming that the water is moving very slowly in the barrel, with what speed does it emerge from the toy? **(7.5 m/s)**



$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

nearly Ø

$$\left(101300 + \frac{2.2\text{N}}{\pi(.005)^2}\right) = 101300 + \tfrac{1}{2}(1000)v^2$$

$$\frac{2.2}{\pi(.005)^2} = 500v^2$$

$$\boxed{v = 7.48\text{m/s}}$$